COMPARATIVE STUDY OF TWO AUTOMATIC REGISTRATION ALGORITHMS

D. Grant[a], J. Bethel[b], M. Crawford[b]

[a] School of Civil Engineering and Geosciences, Newcastle University, NE1 7RU,
darion.grant@ncl.ac.uk
[b] School of Civil Engineering, Purdue University, 47907 - (bethel, mcrawford)
@purdue.edu

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ABSTRACT:

The Iterative Closest Point (ICP) algorithm is prevalent for the automatic fine registration of overlapping pairs of terrestrial laser scanning (TLS) data. This method along with its vast number of variants, obtains the least squares parameters that are necessary to align the TLS data by minimizing some distance metric between the scans. The ICP algorithm uses a "model-data" concept in which the scans obtain differential treatment in the registration process depending on whether they were assigned to be the "model" or "data". For each of the "data" points, corresponding points from the "model" are sought. Another concept of "symmetric correspondence" was proposed in the Point-to-Plane (P2P) algorithm, where both scans are treated equally in the registration process. The P2P method establishes correspondences on both scans and minimizes the point-to-plane distances between the scans by simultaneously considering the stochastic properties of both scans. This paper studies both the ICP and P2P algorithms in terms of their consistency in registration parameters for pairs of TLS data. The question being investigated in this paper is, should scan A be registered to scan B, will the parameters be the same if scan B were registered to scan A? Experiments were conducted with eight pairs of real TLS data which were registered by the two algorithms in the forward (scan A to scan B) and backward (scan B to scan A) modes and the results were compared. The P2P algorithm was found to be more consistent than the ICP algorithm. The differences in registration accuracy between the forward and backward modes were negligible when using the P2P algorithm (mean difference of 0.03mm). However, the ICP had a mean difference of 4.26mm. Each scan was also transformed by the forward and backward parameters of the two algorithms and the misclosure computed. The mean misclosure for the P2P algorithm was 0.80mm while that for the ICP algorithm was 5.39mm. The conclusion from this study is that the symmetric correspondence of the P2P algorithm provides more consistent registration results between a given pair of scans. The basis for this improvement is that symmetric correspondence better deals with the disparity between scans in terms of point density and point precision.

1. BACKGROUND

The registration of terrestrial laser scanning (TLS) data is an essential task in 3D modeling. The data of each scan are inherently referenced to a local coordinate system that is defined by the scanner's setup. Registration is thus needed to obtain a homogeneous dataset from the multiple disparate scans, prior to any 3D modeling and/or analysis.

The development of automatic registration approaches is of interest to many research communities. Among the many automatic approaches that have been proposed, the Iterative Closest Point (ICP) method of Besl and McKay (1992) is one of the most popular methods used. The method is particularly prevalent for the fine registration of pairs of TLS data. Besl and McKay (1992) obtain the least squares parameters that are necessary to align a pair of overlapping TLS data by minimizing the Euclidean distances between points on one scan and their closest points from the other scan.

In the ICP algorithm, the concept of "model" and "data" is used to describe the two unregistered datasets. One of the datasets (the "data") is assumed to be obtained from a digitization process, as done by 3D imaging systems such as terrestrial laser scanners. The other dataset (the "model") is assumed to be an ideal geometric representation of the object being digitized. In this context, registration is performed to align the "data" to the "model". The mathematical formulation thus involves a one-way correspondence search, where corresponding points from the "model" are sought for each of the "data" points.

A similar one-way correspondence approach can be found in the host of ICP variants, for example in the Least Squares 3D Surface Matching (LS3D) method of Akca (2007). Akca (2007) extended the least squares image matching algorithm to incorporate the 3D geometry of point clouds. The LS3D method of Akca (2007) maintained the fundamental least squares image matching concept of a template dataset and search dataset. This template-search concept, as with the model-data concept of Besl and McKay (1992) gives rise to one-way correspondence search. The mathematical formulation involves the correspondences between points from the template dataset and local planes from the search dataset.

Another popular method is the Iterative Closest Plane (ICPlane) method by Chen and Medioni (1991). Although Chen and Medioni (1991) do not explicitly use either the model-data or template-search concepts, their mathematical formulation also involves one-way correspondence. In their point-to-plane approach, Chen and Medioni (1991) selected points from one dataset, and tangent planes are then obtained in the other dataset, as in Akca (2007). Similar usage of one-way correspondences can be found in other ICP variants. The interested reader is referred to Salvi et al. (2007), Bae (2006), Liu (2004) and Rusinkiewicz and Levoy (2001) for related literature.

The model-data concept introduced by Besl and McKay (1992) is appropriate for data obtained in computer vision applications. There are other applications where one of the datasets involved in the registration is in fact an "ideal geometric representation of the object" (Besl and McKay, 1992). However, with TLS data this is not the case as both datasets are obtained from scanning (or digitization). Thus none of the scans can be considered "ideal". The P2P algorithm developed by Grant et al. (2012) recognizes this difference and both scans are treated equally in the registration process. The P2P method utilizes symmetric correspondence, where correspondences are established on both scans.

The importance of the issue of one-way or symmetric correspondence involves consistency of registration parameters. This has not been discussed in the current pairwise algorithms but is of concern, since scans obtain differential treatment in the registration process depending on whether they were assigned to be the model or the data. Consistency of results is not a criterion that is often used to evaluate pairwise registration algorithms. In this paper we investigate this criterion with the ICP and P2P algorithms. Both algorithms are briefly reviewed in the next section. Section three contains a discussion of the experimental results that were obtained from using TLS data acquired at Purdue University. Conclusions and discussion of extensions are included in Section three.

2. OVERVIEW OF ICP AND P2P ALGORITHMS

In this section the algorithms are briefly reviewed, with the emphasis placed on their correspondence approach. Both algorithms are based on the same fundamental mathematical relationship. First, assume that two partially overlapping scans exist, and that the coordinate frames for these scans are different so registration is needed to obtain a homogenous 3D object.

Further, let $\boldsymbol{P}$ and $\boldsymbol{Q}$ refer to the two partially overlapping TLS scans (or surfaces). The two scans are registered to the same coordinate system by using the 3D 6-parameter rigid-body transformation (Cheok, 2006) such that

$$\boldsymbol{Q} = \mathbf{R}(\boldsymbol{P}) + \mathbf{t} \tag{1}$$
$$\mathbf{R} = \mathrm{R}_3(\kappa) * \mathrm{R}_2(\phi) * \mathrm{R}_1(\omega); \qquad \mathbf{t} = [t_x, t_y, t_z]^T$$

$\mathbf{R}$ is the conventional 3D orthogonal rotation matrix formed by 3 sequential rotations $(\mathrm{R}_1, \mathrm{R}_2, \mathrm{R}_3)$ about the x-, y-, and z-axes by the angles ω, ϕ, κ respectively. $\mathbf{t}$ is the vector of translations (t_x, t_y, t_z) that are parallel to the x-, y-, and z-axes respectively. The 6 parameters are thus $\omega, \phi, \kappa, t_x, t_y, t_z$.

The ICP and P2P algorithms both utilize Eq.(1), but correspondences are obtained in different ways.

2.1 The ICP Algorithm

In the ICP algorithm, the model scan is represented by $\boldsymbol{Q}$ in Eq.(1), and the data scan is represented by $\boldsymbol{P}$ in Eq.(1). The closest point in $\boldsymbol{Q}$ is obtained for each point in $\boldsymbol{P}$. Given these set of point-to-point correspondences, the rotation and translation parameters that minimize the mean squared distance between these points are determined. The mathematical expression is given as

$$\underset{\mathbf{R},\mathbf{t}}{\operatorname{argmin}} \frac{\sum_{i=1}^{n} \|\mathbf{Q}' - \mathbf{R}(\boldsymbol{P}) - \mathbf{t}\|^2}{\boldsymbol{n}} \tag{2}$$

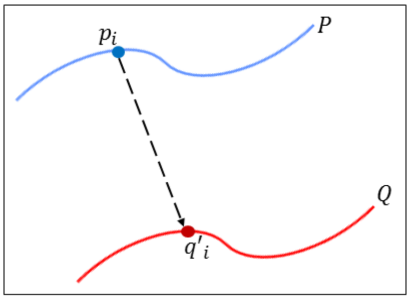

Figure 1. 2D Representation of ICP

In Eq.(2), $\boldsymbol{Q}'$ represents the correspondences for points in $\boldsymbol{P}$ as shown in Figure 1. The ICP algorithm is iterative. Besl and McKay (1992) showed that this algorithm converges monotonically to a local minimum with respect to the mean squared distance objective function. The authors used the quaternion approach for obtaining the least squares parameters. It is important to note that the stochastic properties of the scans are ignored. Also, this approach yields the most accurate results when the points in $\boldsymbol{P}$ are a subset of the model (i.e. $\boldsymbol{Q}$). Besl and McKay (1992) also included an accelerated method in their original work. This method monitors the changes in parameter space and performs extrapolation to help predict the local minimum parameters in fewer iterations.

2.2 The P2P Algorithm

In the P2P approach, Grant et al. (2012) establish point-to-plane correspondences on both scans (see Figure 2). This not only increases the redundancy of the adjustment, but also considers the disparity of both scans simultaneously. This disparity is in terms of the point density and precision. Each point $\boldsymbol{p}_i$ in $\boldsymbol{P}$ is transformed by the current parameters, to obtain $\tilde{\boldsymbol{p}}_i$. The three nearest scanned points to $\tilde{\boldsymbol{p}}_i$ in $\boldsymbol{Q}$ are identified, and the triplet forms its hypothesized corresponding planar element, $(\boldsymbol{q}_e)$, whose normal vector $(\boldsymbol{n}_q)$ is then determined. Similarly, each point $\boldsymbol{q}_i$ is transformed to obtain $\tilde{\boldsymbol{q}}_i$, and its 3 nearest scanned points in $\boldsymbol{P}$ form $\boldsymbol{p}_e$, whose normal vector $\boldsymbol{n}_p$ is determined.

The transformed points $\tilde{\boldsymbol{p}}_i$, and $\tilde{\boldsymbol{q}}_i$ are given by

$$\tilde{\mathbf{p}}_i = \mathbf{R}(\mathbf{p}_i) + \mathbf{t}; \quad \text{and} \quad \tilde{\mathbf{q}}_i = \mathbf{R}^T(\mathbf{q}_i - \mathbf{t}) \tag{3}$$
$$\text{with} \quad \mathbf{p}_i = [p_x, p_y, p_z]_i^T; \text{ and } \quad \mathbf{q}_i = [q_x, q_y, q_z]_i^T$$

Grant et al. (2012) then enforce the signed point-to-plane distances for each correspondence set to be zero, which determines two sets of condition equations. One set relates the correspondences sets in $\boldsymbol{P}$, and another the correspondence sets in $\boldsymbol{Q}$ as follows

$$\boldsymbol{F}_1: (\tilde{\boldsymbol{p}}_i - \boldsymbol{q}_j) \bullet \boldsymbol{n}_{q_i} = 0; \qquad \boldsymbol{q}_j \in \boldsymbol{q}_e$$
$$\boldsymbol{F}_2: (\tilde{\boldsymbol{q}}_i - \boldsymbol{p}_j) \bullet \boldsymbol{n}_{p_i} = 0; \qquad \boldsymbol{p}_j \in \boldsymbol{p}_e \tag{4}$$

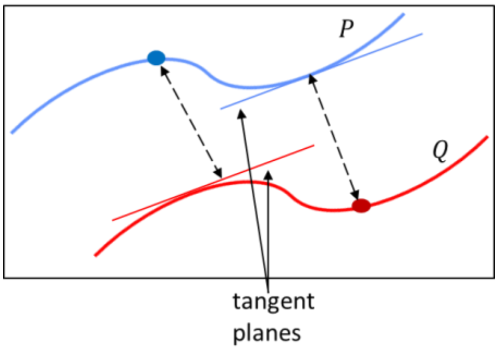

Figure 2. 2D Representation of P2P

The point $\boldsymbol{p}_j$ in Eq.(4) refers to any of the scanned points forming the planar element $\boldsymbol{p}_e$, and similarly for the point $\boldsymbol{q}_j$. In linearized form these two correspondence sets give the classical General Least Squares equation* (Mikhail and Ackermann, 1976)

$$\boldsymbol{A}\boldsymbol{v} + \boldsymbol{B}\Delta = \boldsymbol{f} \tag{5}$$

$\boldsymbol{A}$ is the Jacobian of the condition equations with respect to the observations ($\boldsymbol{P}$ and $\boldsymbol{Q}$),
$\boldsymbol{B}$ is the Jacobian of the condition equations with respect to the registration parameters $(\omega, \phi, \kappa, t_x, t_y, t_z)$,
$\boldsymbol{v}$ is the correction term of observations (residual vector),
Δ is the correction term of the registration parameters (unknown vector),
$\boldsymbol{f}$ is the misclosure term (discrepancy vector).

The parameters are then updated in an iterative fashion by solving the normal equation

$$\boldsymbol{N}\Delta = \boldsymbol{t} \tag{6}$$

$$\boldsymbol{N} = \boldsymbol{B}^T(\boldsymbol{A}\boldsymbol{Q}_{ll}\boldsymbol{A}^T)^{-1}\boldsymbol{B};\ \ \boldsymbol{t} = \boldsymbol{B}^T(\boldsymbol{A}\boldsymbol{Q}_{ll}\boldsymbol{A}^T)^{-1}\boldsymbol{f};\ \ \boldsymbol{Q}_{ll} = \frac{1}{\sigma_0^2}\Sigma$$

$\sigma_0^2 \equiv$ *apriori* reference variance (typically set to 1)
$\Sigma \equiv$ covariance matrix of the observations.

The details of these least squares matrices are provided in Grant et al. (2012) along with the P2P algorithm.

The most significant aspect of the P2P algorithm is Eq.(4), where the same registration parameters (**R** and **t**) are used to establish the correspondences. Thus, one set of parameters (i.e. consistent parameters) is determined between the pair of scans, regardless of which scan is regarded as $\boldsymbol{P}$ or $\boldsymbol{Q}$. It is conceivable that correspondences could be established on both scans as in Figure 2, and two different sets of parameters used. One set for transforming $\boldsymbol{P}$ to $\boldsymbol{Q}$ and one for transforming $\boldsymbol{Q}$ to $\boldsymbol{P}$. This would be analogous to having two one-way correspondence solutions. However, the underlying mathematical relationship as expressed in Eq.(1) describes a pair of scans as having one set of parameters (**R** and **t**). The approach of Grant et al. (2012) is the only algorithm that enforces this condition.

* The General Least Squares adjustment model is also referred to in the literature as the Gauss-Helmert adjustment model, and as the Mixed Adjustment model.

3. RESULTS AND DISCUSSION

Experiments were performed using the Purdue TLS data to investigate the consistency of pairwise registration parameters as determined by the ICP and P2P algorithms. The TLS data were acquired by a Leica Geosystems ScanStation 2. The data included eight scans of the Neil Armstrong statue on Purdue University's campus (see Figure 3 and Table 1). The statue has approximate dimensions of 2.2, 1.8, 2.6 meters (L, W, H). For each scan, the ScanStation 2 was positioned at a distance of 5–10 m from the statue to ensure sufficient overlap. The range and angular precisions of the instrument are 4e-3 meters and 6e-5 radians, respectively.

The ICP and P2P algorithms were implemented in Matlab and compared in terms of registration accuracy, evaluated based on the root mean square error (RMSE) metric. To obtain the RMSE, the registration algorithms were used to estimate the parameters that were needed to register the different pairs of TLS data. Four Leica Geosystem pole targets were also scanned (see layout in Figure 3). The coordinates of these targets were obtained in each scan and a least squares 3D rigid-body parameter estimation was done to determine the transformation parameters for each pair of scans. These parameters were regarded as the reference or ''known'' parameters. The RMSE was then determined for each registration method by computing the root mean square of the Euclidean distances between transformed points using the known parameters, and transformed points using the estimated parameters.

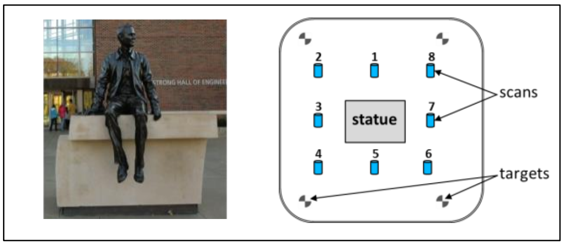

Figure 3. Left: View of Neil Armstrong Statue at Purdue University, from Scan#1. Right: Layout of scans and targets

Table 1. Number of points of each scan of Purdue data.

Scan	1	2	3	4
#Pts	30,128	112,865	76,989	155,717
Scan	**5**	**6**	**7**	**8**
#Pts	157,839	141,630	86,299	104,183

The eight successive pairs of scans were registered. Pair one consisted of scans 1 and 2, pair 2 consisted of scans 2 and 3, and so on. The last pair (pair#8) consisted of scans 8 and 1. The registration was performed in two modes (forward and backward) to investigate the registration consistency. To define the forward and backward modes, consider two scans (1 and 2). In the forward mode, scan 1 is treated as the model (i.e. $\boldsymbol{Q}$) and scan 2 as the data (i.e. $\boldsymbol{P}$) in the ICP and P2P algorithms. Thus, for the forward mode, the parameters that were required to transform the scan with the larger number to the scan with the smaller number were determined. For the backward mode the order was reversed. That is, the parameters to transform scan 1 to scan 2, scan 2 to scan 3 and so on, were determined.

The RMSE values for both the forward and backward modes are shown in Figure 4 and Figure 5, for the ICP and P2P registration methods respectively. Note that the RMSE values for pair#6 and pair#8 by the ICP algorithm were thresholded at 10mm (Figure 4). These values were 16.9mm and 10.7mm respectively. The statistics on the RMSE differences between the two modes are listed in Table 2 for each registration algorithm.

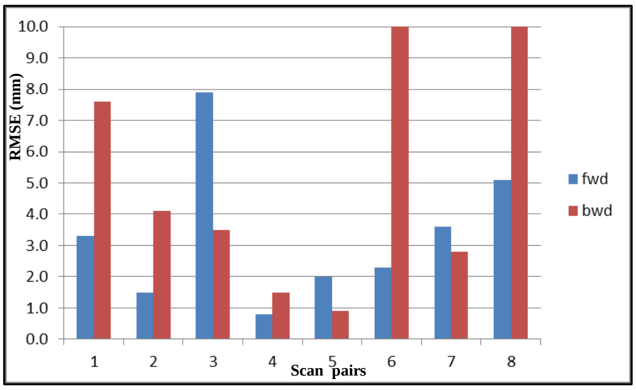

Figure 4. **ICP** registration RMSE (mm) for the eight scan pairs.

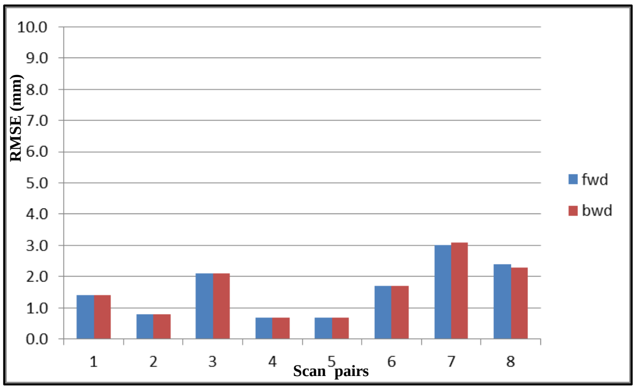

Figure 5. **P2P** registration RMSE (mm) for the eight scan pairs.

Table 2. Statistics on RMSE differences (in mm).

	ICP	P2P
Mean	4.26	0.03
Std	4.57	0.05
Median	3.45	0.00
Max	14.60	0.10
Min	0.70	0.00

The P2P algorithm consistently provided more accurate registration results than the ICP algorithm. More importantly, the P2P parameters that were obtained from the forward and backward modes were consistent with each other, thus yielding near identical RMSE values for all scan pairs. The mean RMSE difference was negligible (0.03mm). For the Besl algorithm however, the variability in the RMSE values was quite large, with a maximum difference of 14.60mm (see Table 2). The mean RMSE difference was 4.26mm.

In Figure 4, the backward mode was the worse overall for the ICP algorithm. However, there is no method for determining which mode (forward or backward) will yield the better results. For three scan pairs, the backward mode yielded the smaller RMSE results in Figure 4, including a two-fold RMSE improvement in scan pair3.

In addition to the RMSE differences, the misclosure for each scan was investigated. To obtain the misclosure the following was done. For scan 2, its estimated registration parameters from the forward mode were used to transform the points to the coordinate frame of scan 1. Then the registration parameters from the backward mode (scan 1 to scan 2) were applied to these transformed points. If the forward and backward registration parameters were identical, each point in scan 2 was returned to its original position. The misclosure is thus the RMSE of the coordinate differences of each point after forward and backward transformations. Figure 6 gives the misclosure RMSE for the ICP and P2P algorithms for each of the eight scans. The statistics on the misclosure are given in Table 3 for each registration algorithm.

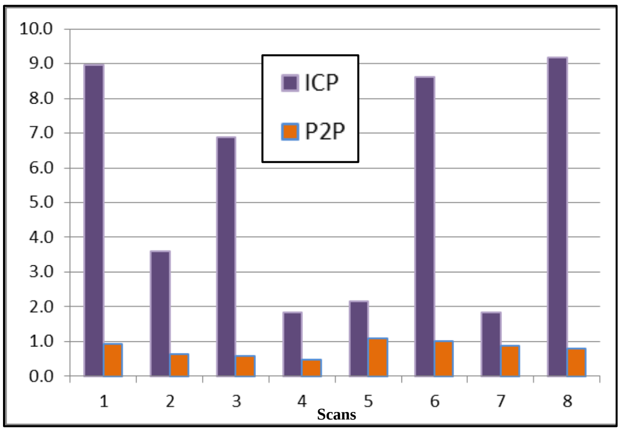

Figure 6. Misclosure RMSE (mm) for the eight scans.

Table 3. Statistics on misclosure RMSE (in mm).

	ICP	P2P
Mean	5.39	0.80
Std	3.35	0.22
Median	5.24	0.84
Max	9.17	1.10
Min	1.84	0.47

The difference between the resulting misclosure from the ICP and P2P algorithms was an order of magnitude. The maximum P2P misclosure was 1.10mm whereas the minimum ICP misclosure was 1.84mm. The mean P2P misclosure was negligible (0.80mm), while that for ICP was 5.39mm.

Both the misclosure RMSE and the registration RMSE show the P2P algorithm to be very consistent in terms of the pairwise registration parameters. This was not the case with the ICP algorithm as the results varied considerable depending on which scan was assigned as $\boldsymbol{P}$ or $\boldsymbol{Q}$. The varied results may be attributed to the fact that TLS registration involves scans that are acquired from different perspectives, which yields point clouds of disparate point density and precision. In the ICP algorithm the differential treatment of the model and data scans (i.e. one-way correspondences) does not adequately deal with these disparities in the registration process.

4. CONCLUSIONS

Cloud-to-cloud pairwise registration involves determining the least squares parameters that minimize some distance metric between two scans. Thus, this registration method can be deemed a least squares adjustment problem. As is the case in all least squares adjustment problems, the results are impacted by the observations that are used. In this paper a study was conducted on two pairwise algorithms, the ICP algorithm by Besl and McKay (1992) and the P2P algorithm by Grant et al. (2012). These two algorithms differ in their correspondence approaches. The ICP algorithm adopts a model-data concept and employs one-way correspondence while the P2P algorithm adopts a symmetric correspondence concept and employs two-way correspondence. In one-way correspondence there is the potential that all points (or observations) will be involved in the least squares adjustment. However, correspondences are only sought for points in one scan ($\boldsymbol{P}$). Essentially, this amounts to using only half of the correspondence sets that are available for the least squares adjustment. In the two-way correspondence approach, all the available correspondence sets are used in the least squares adjustment.

This paper investigated the question of pairwise registration consistency. In other words, consider two scans (A and B), the question to be answered is, will the registration parameters be the same if scan A is registered to scan B and vice versa? The P2P algorithm was found to be very consistent and the differences in registration accuracy between the forward and backward modes were negligible when using the P2P algorithm (mean difference of 0.03mm). However, the ICP had a mean difference of 4.26mm. Each scan was also transformed by the forward and backward parameters of the two algorithms and the misclosure computed. The mean misclosure for the P2P algorithm was 0.80mm whilst that for the ICP algorithm was 5.39mm

The experimental results showed that the ICP yields parameters that can be considerable different depending on which order the scans are used in the registration process. This question of registration consistency is of importance in TLS registration because different scans are acquired from different perspectives which results in point clouds of disparate point density and precision. The ICP algorithm is not sufficient to ensure consistency. The significance of this increases when large numbers of scans are to be registered. It is quite common that series of pairwise registrations be performed to obtain a globally uniform dataset. The quality of the final result will therefore depend on the individual pairwise registrations. Since there is no way of predicting which mode (forward or backward) will yield the best ICP results, then it may require all pairs to be registered in both modes to obtain the best individual pairwise parameters. The P2P algorithm avoids this tedious approach and provides consistent registration parameters based on its symmetric correspondence.

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