PARAMETER ESTIMATION AND MODEL SELECTION FOR INDOOR ENVIRONMENTS BASED ON SPARSE OBSERVATIONS

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ABSTRACT:

This paper presents a novel method for the parameter estimation and model selection for the reconstruction of indoor environments based on sparse observations. While most approaches for the reconstruction of indoor models rely on dense observations, we predict scenes of the interior with high accuracy in the absence of indoor measurements. We use a model-based top-down approach and incorporate strong but profound prior knowledge. The latter includes probability density functions for model parameters and sparse observations such as room areas and the building footprint. The floorplan model is characterized by linear and bi-linear relations with discrete and continuous parameters. We focus on the stochastic estimation of model parameters based on a topological model derived by combinatorial reasoning in a first step. A Gauss-Markov model is applied for estimation and simulation of the model parameters. Symmetries are represented and exploited during the estimation process. Background knowledge as well as observations are incorporated in a maximum likelihood estimation and model selection is performed with AIC/BIC. The likelihood is also used for the detection and correction of potential errors in the topological model. Estimation results are presented and discussed.

1 MOTIVATION AND CONTEXT

Indoor models are of great interest in a wide range of applications. They are of high relevance for indoor navigation, evacuation planning, facility management or guide for the blind. While models of the exterior such as models in level of detail 3 (LoD3) according to CityGML (Gröger and Plümer, 2012) are by now state of the art, modelling of indoor environments is not yet widely available and the acquisition of the corresponding data remains expensive. While most approaches require measurements of high density such as 3D point clouds or images, we propose an approach which gets along with few observations in order to predict floorplans of high accuracy.

The challenge is to estimate an indoor model based on few observations. More precisely, the task that we consider in this paper is to place a set of n rectangular rooms within a polygonal footprint, locating the doors and estimating the height of the rooms. The resulting model is characterized by discrete and continuous model parameters that are related by both linear and non-linear constraints. Our approach is characterized by sparse observations such as room areas, functional use, window locations and, possibly, room numbers that can be acquired from building management services. Indoor images or laser scans are not required. A maximum a-posteriori (MAP) estimation uses further statistical knowledge such as probability density functions and Gaussian mixtures for model parameters.

In this paper, we assume that the topological model is provided by a preceding step using Constraint Logic Programming as presented by Loch-Dehbi et al. (2017). The topological model consists of the room neighbourhood information and the correspondence between rooms and windows. This article focuses on the optimal estimation of model parameters. Therefore, we use a Gauss-Markov model with Bayesian background knowledge. We represent and exploit symmetries and repetitive structures. This contributed to improving the existing approach and thus the estimation of model parameters. We asses the quality of the model by a maximum likelihood estimation and compare different models by the information criteria Bayesian information criterion (BIC) and Akaike information criterion (AIC). This allows for the comparison of model hypotheses based on their likelihood.

The main contribution of this paper is the improved estimation of model parameters based on a-priori derived topological models performed by Loch-Dehbi et al. (2017). This includes:

- the use of prior knowledge in the form of probability density functions for model parameters,
- an MAP-estimation considering observations such as the area of rooms, the location of windows and the building foot-print,
- the representation and exploitation of symmetries, especially translational ones,
- detection and correction of errors in the topological model,
- calculation of normalized likelihoods for each model hypothesis and comparison of models with a derived ranking.

All in all, for the prediction and estimation of a floorplan and its parameters with high accuracy, the areas of the rooms, the footprint and the location of windows together with a prior knowledge in form of probability density functions are sufficient.

The remainder of this article is structured as follows. The next section discusses related work. Section 3 introduces the necessary theoretical background of the used stochastic approach. Section 4 gives insight into the derivation of topological models using constraint satisfaction. Section 5 explains our stochastic estimation reasoning in detail, while Section 6 discusses the experimental results. The article is summarized and concluded in Section 7.

2 RELATED WORK

Indoor models are gaining more and more attention for many tasks such as rescue management. Turner and Zakhor (2014) generated building floorplans from laser range data based on a triangulation of a 2D sampling of wall positions. Becker et al. (2015) proposed a grammar-based approach for the reconstruction of 3D indoor models from 3D point clouds. Yue et al. (2012) used shape grammars describing a building style together with a footprint and a set of exterior features to determine the interior layout of a building. Khoshelham and Díaz-Vilariño (2014) applied also shape grammars for 3D indoor modelling by iteratively placing, connecting and merging cuboid shapes. However, a pre-design of the grammar rules is required. Ochmann et al. (2016) segment a point cloud into rooms and outside area for reconstructing a scene using an energy minimization in a labelling problem. In contrast to our approach, all mentioned approaches require dense observations such as 3D point clouds from laserscans or range cameras using mobile mapping systems that are both cost and time expensive. Measurements are expensive because each single room has to be accessed.

Various works tried to overcome the measurement overhead by using low cost sensors. To this end, Rosser et al. (2015) presented a method for constructing as-built plans of residential building interiors. The prediction is based on mobile phone sensor data and improved by incorporating hard and soft constraints. Pintore et al. (2016) generate 2.5D indoor maps based on images acquired by mobile devices. Diakité and Zlatanova (2016) used the low cost Android tablet from Google's Tango project for the acquisition of indoor building environments. However, it is not possible to derive detailed indoor models due to artefacts and anomalies in the sensor data. Furthermore, the derivation of models from measurements is difficult due to the masking of walls by furniture. As a consequence, our main contribution is the parameter estimation and model selection for unknown substructures in buildings such as floorplans based on only few observations like the area of rooms and footprints. We make use of strong model assumption supported by a profound background knowledge.

In our context, Rosser et al. (2017) demonstrated a two-staged semi automatic data-driven estimation of 2D building interior floorplans based on limited prior knowledge. However, their approach requires beforehand topology of rooms and the orientation of one room. Compared to Rosser et al. (2017), in our approach the topology is automatically provided using Constraint Logic Programming. Apart from stochastic reasoning, our approach draws upon ideas of constraint satisfaction. A constraint-based approach which generates possible floorplans respecting a set of geometric constraints is presented by Charman (1994). This approach does not address as-built models and, however, does not consider probable configurations. An overview of works in in-door modelling and mapping is given by Gunduz et al. (2016).

3 GAUSS-MARKOV MODELLING OF FLOORPLANS

The estimation of the location and shape parameters of floorplans has been addressed as a reasoning process which combines constraint propagation and a maximum a-posteriori estimation based on special graphical models (Loch-Dehbi et al., 2017). The performed MAP-estimation is based mainly on the background knowledge about room location and shape parameters. In order to acquire more accurate estimations, observations such as window locations have to be integrated in the model selection as well. Furthermore, regularities characterizing man-made objects such as buildings are beneficial and have to be exploited. This paper focuses on the integration, modelling and assessing of the impact of these aspects on the estimation process. Hence, we provide a method improving the mentioned MAP-estimation.

To this aim, based on a predetermined topological floorplan model consisting of the neighbourhood of rooms and their window correspondence, the task we solve can be addressed as a stochastic parameter estimation defined in the form of a Gauss-Markov model as follows:

$$\tilde{l} = l + \tilde{v} = A\tilde{x} + a, \tilde{v} \sim M(0, \Sigma_{ll}) \tag{1}$$

where l refers to an observation vector which can deviate from the true values \tilde{l} of the observations with residuals \tilde{v} . \tilde{x} denotes the true values of the unknowns, i.e model parameters, while Astands for the design matrix mapping the model parameters onto the observations. By uncertain observations, a distribution Mis introduced. Our task lies in finding estimates \hat{x} of the true model parameters \tilde{x} from the observations l. In our context, the parameters as well as the residuals are a-priori unknown which leads to an underconstrained problem. This can be overcome by requiring that the weighted sum of the squared residuals

$$\Omega(x) = v^T(x)\Sigma_{ll}^{-1}v(x), \qquad (2)$$

of the residuals v(x) = Ax + a - l has to be minimal as follows:

$$\widehat{x} = \operatorname{argmin}_{\mathbf{x}} \Omega(x) = (A^T \Sigma_{ll}^{-1} A)^{-1} A^T \Sigma_{ll}^{-1} (l-a).$$
(3)

In the case of a normal distribution M, this is equivalent to determining the maximum likelihood estimation of the parameters:

$$p(l \mid x) = \frac{1}{(2\pi)^N |\Sigma_{ll}|^{1/2}} \exp\left(-\frac{1}{2}v(x)^T \Sigma_{ll}^{-1} v(x)\right).$$
(4)

N is the rank of the covariance matrix Σ_{ll} . Since our problem is also characterised by bi-linear dependencies such as it is the case between the room areas as multiplication of room shape parameters, the modelling of non-linear constraints has to be taken into account. In this context, the linear problem is expanded by a non-linear part modelled by non-linear Gauss-Markov model as follows:

$$\underline{l} \sim M(f(\tilde{x}), \Sigma_{ll}), \tag{5}$$

where the parameters are related by a twice differentiable function f. In this case, the goal is to find an estimation:

$$\widehat{x} = \operatorname{argmin}_{\mathbf{x}} \left(l - f(x) \right)^T \Sigma_{ll}^{-1} (l - f(x)), \tag{6}$$

which can be solved by Taylorization in the usual way (Niemeier, 2008) starting from approximate values of the unknown model parameters. For more details, the interested reader is referred to Förstner and Wrobel (2016).

Given the mathematical model, we are now interested in choosing one model among several alternatives taking some observations linto account. In our context, the observations consist of the locations of windows, room areas and the footprint geometry. To this aim, a Bayesian approach is used in order to select a model Mhaving the largest posterior probability $P(M \mid l)$ out of several models M_m :

$$\widehat{M} = \operatorname{argmax}_{m} p(l \mid M_m) P(M_m).$$

The model is then defined following the Akaike information criterion (AIC) (Akaike, 1974) according to equation (7).

$$M_{\rm AIC} = \operatorname{argmax}_{m} - \log p(l \mid M_m) + U_m.$$
(7)

 U_m denotes the number of the parameters x_m of the model M_m characterizing the model complexity. By normally distributed data the first term in equation 7 can be written as:

$$-\log p(l \mid M_m) = \frac{1}{2} \widehat{v}^T \Sigma_{ll}^{-1} \widehat{v}.$$
 (8)

The model M_m can be likewise determined using the Bayesian information criterion (BIC) (Schwarz et al., 1978) augmenting the term U_m by the number N of the observations as following:

$$M_{\rm BIC} = \operatorname{argmax}_{\rm m} - \log \ p(l \mid M_m) + \frac{1}{2}U_m \log N.$$
 (9)

4 CONSTRAINT PROPAGATION FOR TOPOLOGICAL FLOORPLAN DERIVATION

This section gives an overview about the method for deriving topological floorplan models as prerequisite for the subsequent stochastic reasoning. To solve the presented non-linear problem with discrete and continuous parameters, we have principally to cope with an a-priori infinite continuous search space. However, we proposed a method in which the search space is narrowed using architectural constraints and browsed by intelligent search strategies using domain knowledge.

We start with solving a discrete combinatorial problem that yields a good initialization for the stochastic reasoning. The correspondence of a-priori known windows to their rooms is unknown. The bilateral relations between rooms have to be determined as well. This consists in the knowledge about vertical and horizontal neighbourhood of rooms leading to a topological model. In order to make the combinatorial problem of locating rooms within the footprint feasible, we build upon background knowledge consisting in probability density functions (PDFs) of shape and location parameters learned from extensive training data. We applied Gaussian mixtures as a good approximation to model skew symmetric or multi-modal distributions, e.g. PDFs. Using Expectation Maximization (McLachlan and Peel, 2000), a Gaussian mixture of *m* components each weighted by its probability ω_i for each continuous parameter is estimated for the reasoning process:

$$\sum_{i=1}^{m} \omega_i N(\mu_i, \sigma_i^2). \tag{10}$$

This allows for using well established stochastic reasoning afterwards.

We exploit architectural regularities and design corresponding constraints on model parameters to restrict the search space. For instance, it is not allowed that walls cross windows. Besides, a lower window for example is part of an *i*th room if the y-coordinate of the room's lower left corner y_i equals the width of the outer wall and the following constraint is satisfied:

$$((x_i \le wiLoc_{j,1}^l) \land (wiLoc_{j,2}^l \le (x_i + width_i))), \quad (11)$$

where $wiLoc_{j,1}^{l}$ and $wiLoc_{j,2}^{l}$ are the location parameters of a lower window placed in the *j*th room as shown in Figure 2. The constraint 11 implies in this case j = i. The single components of the Gaussian mixture define upper and lower bounds for the according continuous parameter. For example, the model parameters of the room, that is its lower left corner (x_i, y_i) and its width



Figure 1: Topological floorplan model derived using constraint propagation

and depth, are restricted by upper and lower bounds in addition to shape parameters of the corresponding footprint.

Since the floorplan model can be described by constraints on discrete as well as continuous variables with associated domains, it can be addressed as a constraint satisfaction problem. By incorporating consistency-enforcing algorithms and constraint propagation, existing constraints are tightened or new constraints are derived to narrow the search space (Dechter, 2003).

Figure 1 shows one topological model as a result of our constraint propagation using Constraint Logic Programming (Frühwirth and Abdennadher, 2003). It yields the bilateral relations between rooms, the correspondence of windows to rooms and the selection of single components of the Gaussian mixtures for the model parameters. We ignored temporarily the walls and relaxed the search problem in order to provide buffers for constraint propagation with discrete domains. Consequently, the predicted preliminary rooms do not fill the entire footprint space. Particularly, the alignment of rooms along a corridor is not guaranteed. The acquired topological model serves as input for the parameter estimation and model selection as presented in the following section. For more details on this combinatorial step the reader is referred to Loch-Dehbi et al. (2017).

5 PARAMETER ESTIMATION AND MODEL SELECTION OF FLOORPLANS

This section describes our approach for parameter estimation and model selection for indoor models based on sparse observations. Based on preliminary topological models acquired from a reasoning process following the method described in Section 4, an optimal parameter estimation using a Gauss-Markov model and Bayesian model selection is performed. The detailed method is described by Loch-Dehbi et al. (2017). We exploit the symmetries, especially translational ones, characterising buildings in order to improve the estimation of model parameters. In this sense, the distances between windows and walls as well as distances between windows themselves are considered. These kinds of symmetries are modelled in an explicit way. Models including symmetries and those without symmetries are taken into consideration and compared as alternative models.

Our approach starts with defining the complete Gauss-Markov model which consists of the functional and stochastic model including the design matrix A according to Equation 1. In our sce-



Figure 2: Illustration of the used model parameters and observations.

nario, the additive term a is equal to zero. The model components are elaborated in the following.

Functional model:

We assume that the adjacencies between the rooms and the respective windows as well as the ordering among the rooms are given by the topological model as described above in Section 4. The exact location and shape parameters are not required. In this case, the functional model is uniquely specified by the following model parameters:

- the widths and the depths of the lower (width^l_i, depth^l_i) and upper (width^u_i, depth^u_i) rooms. The index i refers to the ith room. As we assume that the rooms are aligned along corridors, we have two single parameters (depth^l, depth^u) both for the depth of the lower and the upper rooms, but distinct parameters for the width of each room.
- two parameters for the width and the depth of the corridor (*width*^{corr}, *depth*^{corr}).
- two parameters for the width of the walls. Thereby we assume that the widths of all outer walls (*waOut*) are the same, as are the inner walls (*waIn*).
- two parameters for the width and the depth of the floorplan which is assumed to be rectangular and, above, having its lower left corner in the origin of the local coordinate system. Thus they are named x_{\max} and y_{\max} .
- The area $area_i$ for each *i*th room and for the corridor.

Although it is not necessary to identify a model in a unique way, we introduce the following parameters in order to relate the model to the given observations:

 the location of the lower (wiLoc^l_{i,1}, wiLoc^l_{i,2}) and upper (wiLoc^u_{i,1}, wiLoc^u_{i,2}) embrasures of the windows in the *i*th room. We need only those embrasures which are directly adjacent to a wall.

Figure 2 gives an overview of the domain model and its parameters. Following a Bayesian approach, we explicitly incorporate prior knowledge and represent it by pseudo-observations following the ideas of Förstner and Wrobel (2016) and Bishop (2007). Note that prior knowledge on model parameters is represented by Gaussian mixture distributions and that we have already selected the according component of each parameter from these distributions in a preceding step. Thus, in this paragraph we can assume that model parameters are normally distributed with known means μ and variances σ . We will use pseudo-observations for the following model parameters:

- room widths $width_i$ and depths $depth_i$,
- corridor width *width*^{corr} and depth *depth*^{corr},
- outer and inner wall widths waOut and waIn.

Design matrix:

We apply the Gauss-Markov model where each observation is a linear function of the model parameters. Since the Gauss-Markov model is generic and the relation between model (parameters) and observations is explicit, it can be used both for simulations and parameter estimation. Thus, it is appropriate for the purpose of this article.

For the design matrix we have linear and bi-linear constraints. We start with the linear constraints. For a left embrasure in an *i*th lower room, we have constraints which relate this observation to the left adjacent wall:

$$waOut + (i-1)waIn + \sum_{j=1}^{i-1} width_{j}^{l} + wiDist_{i,1}^{l} = wiLoc_{i,1}^{l}$$
(12)

Likewise, we have for the right embrasures constraints which relate them to the right adjacent wall:

$$waOut + (i-1)waIn + \sum_{j=1}^{i} width_{j}^{l} - wiDist_{i,2}^{l} = wiLoc_{i,2}^{l}$$
(13)

The constraints for the embrasures in the upper rooms are formulated in an analogous way. The depth of the floorplan y_{max} is related to the depth of the upper and lower rooms including the corridor as well as the width of the respective walls. Since we assume that rooms are aligned, we have a single depth parameter for the upper and the lower row of rooms respectively:

$$2waOut + 2waIn + depth^{l} + depth^{u} + depth^{corr} = y_{\max}$$
(14)

For the sequences of upper and lower rooms, we have two constraints which adds up their widths together with the widths of the adjacent walls. For instance, the constraint for lower rooms is formulated as follows:

$$2waOut + (r-1)waIn + \sum_{i=1}^{r} width_{i}^{l} = x_{\max}, \qquad (15)$$

where r refers to the room number in a room sequence. A similar constraint holds for the corridor and its adjacent walls:

$$2waOut + width^{corr} = x_{\max}$$
(16)

Note that there seems to be an incoherence between the last three constraints and the Gauss-Markov model which assumes that there is a dependency between model parameters and each single observation. Here, the width x_{\max} of the floor is used as observation twice. We can, however, argue that in fact we have two observations, which have the same value only under the assumption that the shape of the floorplan is rectangular. Since we will assume a very low variance for x_{\max} , duplicating this value will be of no

harm for the parameter estimation. Table 1 summarizes the used model parameters, observations and pseudo-observations as well as the used abbreviations accordingly.

Each pseudo-observation gives one linear constraint which directly relates the model parameter to the mean μ_k of the *k*th selected component of the Gaussian mixture for the respective parameter. For room width for example we get:

$$width^u = \mu_k^{width}.$$
 (17)

The constraints for the room areas are defined as follows:

$$width_i \times depth_i = area_i,$$
 (18)

these constraints have a bi-linear form. Non-linearity is handled by Taylorization using partial derivatives. This leads to the linear constraints of the form:

$$depth_i width_i + width_i depth_i = \Delta area_i.$$
(19)

Note that in this equation the underlined terms are coefficients in the design matrix while the other terms refer to the respective model parameters. The coefficients stem from a starting vector x_0 . The selection of a good start vector is important. To derive this vector there are several options. As a first choice prior knowledge, i.e. pseudo-observations could be used. An alternative would be the parameter values of the topological model. We found that a better choice is the estimation of these parameters using a relaxed Gauss-Markov model with a design matrix where the non-linear constraints are omitted. This approach derives good estimations for model parameters up to the width of the upper and lower rooms and the floor widths. Note that - beside the bi-linear constraints for the areas - these parameters are related by one single constraint (cf. Equation 14). For this reason, the observed room area of the *i*th room and the estimated width $width_i$ is used to derive the two medians of the widths of the aligned lower and upper rooms:

$$depth_i = \text{median}\left(\left\{i \mid \frac{area_i}{width_i}\right\}\right).$$
 (20)

Stochastic model:

Observations are assumed to be uncorrelated random variables with normal distributions. The following standard deviations σ are assumed:

- embrasures: 0.1 m
- room areas: 0.1 m^2
- x_{max} and y_{max} : 0.01 m
- interior and exterior walls: 0.03 m

For the pseudo-observations of room depths and widths, standard deviations of 1 m are assumed. It was noted that the difference between pseudo-observations and assumed model parameters exceed the standard deviations.

Parameter estimation:

In comparison to the models derived by Loch-Dehbi et al. (2017), the MAP-estimation includes both prior knowledge in the form of probability density functions of model parameters and observations such as window locations and room areas. Furthermore, the representation and exploitation of symmetries enabling a more

	$width_i^l$	lower room width			
model parameters	$width_i^u$	upper room width			
	$depth_i^l$	lower room depth			
	$depth_i^u$	upper room depth			
	$width^{corr}$	corridor width			
	$depth^{corr}$	corridor depth			
	waOut	outer wall width			
	waIn	inner wall width			
	$depth^l$	lower depth of rooms			
	$depth^u$	upper depth of rooms			
	in d				
	$wiDist_{i,1}^{i}$	lower dist(left embrasure,wall)			
	$wiDist_{i,2}^{\iota}$	lower dist(right embrasure,wall)			
	$wiDist_{i,1}^u$	upper dist(left embrasure,wall)			
	$wiDist^u_{i,2}$	upper dist(right embrasure,wall)			
observations	$wiLoc_{i,1}^l$	left lower embrasure right lower embrasure left upper embrasure			
	$wiLoc_{i,2}^{l}$				
	$wiLoc_{i,1}^{u}$				
	$wiLoc^{u}_{i,2}$	right upper embrasure			
	$area_i$	room area			
	$area^{corr}$	corridor area			
	x_{max}	floor width			
	y_{max}	floor depth			
pseudo- observations	μ_k^{width}				
	$\mu_k^{width^{corr}}$	man of the 1th Coursian			
	depth	mean of the kth Gaussian			
	, depth ^{corr}	mixture component of a parameter			
	μ_k				
	μ_k^{walk}				
	$\mid \mu_k^{waOut} \mid$				

Table 1: Overview of the domain parameters, observations and pseudo-observations for floorplan model selection. The index i refers to the *i*th room.

accurate estimation is taken into account. The influence of the occurrence of repetitive patterns and structures such as the same window distances to the next wall or recurrent room width has been investigated in a case study. We performed a simulation using Gauss-Markov modelling and taking these aspects into consideration and were able to compare the impact of such patterns on the resulting estimations.

All in all, the mentioned approach contributed to the improvement of the accuracy of the estimated model parameters. Table 2 summarizes the achieved accuracies under different simulated conditions. For the estimation of room widths, for instance, an average accuracy of about 1.32 cm is achieved using a bi-linear model and assuming that the distances between the window embrasures and the next left or right wall are fix. In this case, the estimated and the ground truth model are almost identical as depicted on the right of Figure 3. In contrast, we can state worse accuracies without taking repetitive structures such as room widths into account using a linear model. So far we have assumed that adjacent rooms are aligned along the corridor (i.e. they have the same width) and inner and outer walls respectively have the same widths. In reality in many cases indoor models reveal much more symmetries. In this paper, we focus on two translational symmetries:

- distances between walls and neighbouring window embrasures are equal
- the widths of rooms are either equal or equal up to a small integer factor (two or three).

Interestingly, parameter estimation derived so far provides a good basis for the identification of such symmetries. The covariance

	room width	room depth	inner wall	outer wall	corridor width	corridor depth
linear model	68.2	62.73	2.33	2.43	4.9	63.46
bilinear variable window distances	5.75	1.2	1.66	1.83	3.73	0.96
bilinear fixed window distances	1.32	1.14	1.36	1.51	3.14	0.86
bilinear room symmetries	0.41	1.1	1.65	1.82	3.72	0.96

Table 2: Average deviations between the estimated shape parameters of rooms, walls and corridors and the true parameters in [cm].



Figure 3: Floorplan estimation using our approach. Model selection is performed based on linear (top) and bi-linear (bottom) constraints. Symmetries and repetitive patterns (fixed room widths, fixed window distances) are considered. The zoomed part on the right shows that the estimated (orange) and the ground truth model (black) are almost identical

matrix of the estimated model is given by:

$$\Sigma_{\widehat{x}\widehat{x}} = (A^T \Sigma_{ll}^{-1} A)^{-1}.$$
 (21)

Its diagonal gives an estimation of the accuracy of the parameter estimation. Comparison of these accuracies with the differences of related model parameters such as room widths or distances between walls and embrasures provides hints on symmetries. One could now use a classical hypothesis test to check for identity. Instead, we have represented symmetries in our models explicitly and made model comparisons with AIC or BIC as mentioned in Section 3 (cf. Equations 7 and 9).

Let *n* be the number of rooms. If symmetries w.r.t distances between walls and embrasures are explicitly represented, the 2nparameters for distances are replaced by one single model parameter, and the number of columns of the design matrix is reduced accordingly. Please note that the number of observations and thus the number of rows of the design matrix is not changed. Thus, in principle the respective likelihoods (cf. Equation 4) can be compared and used for model selection. The complexities of the respective models are however different, reflected by a different number of model parameters. That is the reason why penalty terms as in the Akaike information criterion (Equation 7) or the Bayesian information criterion (Equation 9) have been applied. In our case both criteria led to the same results. To assess to what extent one model outperforms the other normalized likelihoods (pseudo-probabilities) were used:

$$LH_n(i) = \frac{LH(i)}{\sum_{j=1}^n LH(j)}.$$
(22)

In order to represent identity of room widths (up to a factor k) in the model and the corresponding design matrix explicitly, again the n parameters for room widths are replaced by one single model parameter. In the equations the term $width_i$ is replaced by $k_i \times width$ where k_i is integer depending of the specific *i*th room. Again the number of columns of the design matrix is reduced considerably. But since the prior knowledge of this model parameter is represented by pseudo-observations, the number of rows is reduced as well. Thus, the likelihoods given by Equation 4 are not comparable immediately. To make them comparable the covariance matrix Σ_{ll} is projected to a sub-matrix Σ'_{ll} where the rows for the pseudo-observations for room widths are omitted. Differences in model complexities are again reflected in the penalty terms of AIC and BIC, and normalized likelihoods are derived.

Figure 3 shows the results of the floorplan estimation using our approach. As mentioned, the model estimation and selection is performed based on linear (first row) and bi-linear (second row) constraints. The impact of symmetries such as recurrent room widths or repetitive window distances is investigated. The zoomed part on the right reveals that the estimated (orange colour) and the ground truth model (black colour) are almost identical.

We have also studied how deficiencies of the topological model can be identified and corrected by the stochastic parameter estimation. Figure 4 illustrates a situation of the floorplan of our institute in a Wilhelminian style building where a small room (in orange) of 1.8 m^2 is contained in another room. This violates the assumption that rooms are rectangular – the outer one is not. Let us assume that the combinatorial reasoning deliberately excludes rooms with an area under a given threshold (e.g. $2 m^2$) and leaves the problem to the stochastic reasoner. The stochastic reasoner as described above starts with an observational error for the area of that specific room. Having in mind that one small room of a known size has been omitted, the task is now to identify the room which is affected and to correct the observed area by the sum of the areas of the enclosed room and the enclosing room. If this leads to a model with a considerably larger likelihood, it can be inferred that the smaller room is enclosed in or adjacent to the other (although still unclear exactly how). We have studied such scenarios and used the covariance matrix of the fitted observations:

$$\Sigma_{\widehat{i}\widehat{i}} = \left(A\Sigma_{\widehat{x}\widehat{x}}A\right)^T,\tag{23}$$

which has often been used for the identification of outliers in the observations. Surprisingly this did not lead to satisfactory results. Instead, we applied the same procedure as given above and used respective likelihood for model comparison in order to identify the affected room and to correct the area observation accordingly.

6 EXPERIMENTAL RESULTS

Our approach starts with defining the functional model and a start vector \tilde{x} of true model parameters according to Figure 2 and the stochastic model as described above. The stochastic model together with the assumption of normal distributions was used to generate observational errors \tilde{v} in order to generate noisy observations $l = l + \tilde{v}$. We performed this for 1000 test cases and derived estimated model parameters \hat{x} for the different scenarios described above. We compared the estimated values \hat{x} with the true values \tilde{x} . The average deviations are listed in Table 2. Obviously, the transition from the linear in the first row to the bi-linear model in the second row leads to a considerable improvement of the estimation of room width and room depth. That, however, is not really surprising since room area is not considered in the linear model. However, the improvement of model accuracy by the introduction of further model assumptions is surprising. If we leave aside corridor width for a moment, both the inclusion of symmetries of window distances and of room width symmetries leads to accuracies in the range of 1 cm. This should be compared to observational accuracy of window locations of 10 cm which illustrates the important role of symmetries for model parameter estimation.

To get a different impression of this fact, one can look at the redundancies of the model. Let us again assume n rooms and – to simplify discussion - the same number of windows. If we omit the window locations for the moment and restrict to the essential model parameters, we have n + 6 model parameters (n room widths, 2 depths, 1 corridor width and 1 corridor depth and 2 walls) and n + 5 observations (n room areas, 1 corridor area, $3 \times x_{max}$ and $1 \times y_{max}$). This means that we have an underconstrained system. If we introduce window locations we get 2n additional observation and 2n additional model parameters, which leads again to an underconstrained system. To achieve an overconstrained system with a certain amount of redundancy, pseudoobservations have been used. Whereas the number of model parameters remains the same, we now get 2n + 8 observations. This means, however, that prior knowledge on the distribution of model parameters plays a dominant role. This role is considerably reduced by the explicit representation of symmetries. Window distances lead to a reduction of 2n-1, while room widths contribute to a reduction of n-1. Interestingly, both cases lead to similar results with regard to accuracies.

We also studied how sure we can be with regard to the assumption of symmetries. As described above, we used AIC and BIC and derived normalized likelihoods. In all cases, we found that with 0.99 versus 0.01 or better the true symmetric model outperformed the model without this assumption considerably.

In the current setting, we assume that except for corridors each room is associated to at least one window. More general settings allow for having rooms without windows. As yet, our approach described how to derive floorplans from sparse observations. The 2D models can be extended following the ideas presented by Loch-Dehbi et al. (2017). Especially door shapes and



Figure 4: Detection and correction of potential errors in the topological model. The small room omitted from the topological models is estimated using stochastic reasoning.

positions are predicted based on probability density functions approximated by Gaussian mixtures. Prior knowledge about windows is exploited to localize the floor within the considered building. As described by Dehbi et al. (2016), the height of each storey is derived by kernel density estimations based on 3D point clouds of façades stemming from an unmanned aerial vehicle. These conditional predictions depend on a given building style, otherwise the latter can be inferred following the ideas of Henn et al. (2012).

7 CONCLUSION

This paper presented an approach for predicting and reconstructing a-priori unknown structures in building interiors. In contrast to common approaches, the presented work does not rely on dense observations such as 3D point clouds. For the prediction of a floorplan, the areas of the rooms, the footprint and the location of windows together with a prior knowledge in form of probability density functions is sufficient to estimate model parameters with high accuracy.

Symmetries and other regularities in man-made objects allow for a top-down process considering an indoor model with strong constraints and regularities. The latter is however characterized by linear and bi-linear relations with discrete and continuous parameters. An extensive analysis of a ground truth database yielded a profound prior knowledge that supported the estimation of floorplans. This includes distributions of model parameters that are characterized by probability density functions and approximated by Gaussian mixtures.

In this paper, we focused on the improvement of model parameters in an a-priori estimated topological model that was generated by the use of Constraint Logic Programming. Our approach uses a Gauss-Markov model and incorporates not only probability density functions, but also observations such as window locations or room areas. It further benefits from the representation of symmetries in the known exterior model. Bayesian model selection is based on the information criteria AIC and BIC and incorporates observations in order to choose the hypotheses that best fit the scenario. Errors in the preliminary topological model are detected and corrected. Thus, accuracies in the range of 1 cm are possible. The presented approach provides a ranked set of model hypotheses with corresponding normalized likelihoods.

In this paper, we assume that rooms as well as the corresponding footprint have a rectangular shape following a Manhattan world assumption. More general room layouts such as L, T or even U shaped rooms will be subject of future work. The considered floorplans characterize office buildings. Model selection for other building types like housing, where room numbers are not given, will have to use slightly different models. The derivation of the functional model from the topological model will be addressed in a subsequent article. Our approach does not rely on dense observations such as 3D point clouds from laserscans or range cameras. However, additional observations of model parameters stemming from such sensors can be integrated easily. They will improve the accuracy of the estimated hypothesis.

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