

QUANTITATIVE ASSESSMENT OF THE PROJECTION TRAJECTORY-BASED EPIPOLARITY MODEL AND EPIPOLAR IMAGE RESAMPLING FOR LINEAR-ARRAY SATELLITE IMAGES

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ABSTRACT:

Epipolar geometry rectification is one of the critical issues in photogrammetry, which is a strong corresponding searching constraint in dense image matching for 3D reconstruction. In this paper, the properties of the projection trajectory-based epipolarity model are analyzed quantitatively, and the approximate straight line and parallelism property of the epipolar curve are discussed comprehensively using the linear pushbroom satellite images, i.e. IKONOS, GeoEye images. Based on the analysis of the epipolar line properties, a practical method for epipolar resampling developed. In this method, the pixelwise relationship is established between the original and the epipolar images. The experiments on TH-1 images show that quasi rigorous epipolar images can be resampled using our proposed method for both along-track images. After epipolar geometry rectification, the vertical parallaxes at checkpoints can achieve sub-pixel level accuracy, thus demonstrating the correctness and applicability of the proposed method.

1. INTRODUCTION

Linear-array pushbroom imaging sensors have been the leading payload of current high-resolution optical satellites. As full used sensors, it has fueled applications in a variety of domains, including geolocation, digital surface model (DSM), 3D reconstruction and mapping by delivering satellite images and geolocation models (i.e. a rigorous model or a Rational Polynomial Coefficients (RPC) model as a polynomial approximation from different satellite image vendors) (Y. Zhang et al., 2004).

Since the geometry of the linear pushbroom imaging sensors model is more complicated when comparing with the collinearity equations of the perspective cameras. The methods of the traditional frame perspective imaging model are no longer universally applicable for the satellite images. Linear-array pushbroom images are characterized "line of central projection" dynamic imaging and the exterior orientation elements vary for each scanning line. There is no rigorous definition of the epipolar line as the frame perspective image. Moreover, the coverage area of high-resolution satellite images is much larger than aerial or street-view images, which results in a vast number of pixels (billion-level pixels). This is how the considerable gap between the photogrammetry and computer vision was created over the past few decades, which should be bridged to benefit each other in scientific development. Previous studies analyzed less on the geometric property of the epipolarity model or the central projection models in the aerial images are adopted directly. Research on satellite image epipolarity model and epipolar geometry rectification method has been one of the hot topics in the field of aerospace photogrammetry and remote sensing.

In the early 1980s, Z. Zhang and Zhou (1989) proposed a method for generating an approximate epipolar line based on polynomial

fitting of the corresponding point coordinates for the SPOT across-track stereo images. Kim (2000) analyzed the projection trajectory (PT) method based on a rigid model. It has been clear about the epipolar curve shape; at the same time, it concluded the approximate linear characteristic of the epipolar curve within a certain range, which laid a foundation for the subsequent application. Morgan et al. (2006) and Habib et al. (2005) proposed a method for generating an approximate epipolar line based on a parallel projection model, according to the small imaging field angle characteristics of high-resolution satellite imagery. The sensor model based on parallel projection was adopted for satellite image, then the epipolar line model in the form of a straight line was proposed based on at least four pairs of corresponding points. Wang et al. (2011) obtained the direction of the epipolar line on the reference plane by the projection trajectory method and projected along the epipolar line direction to get an approximate epipolar line.

By adopting this method, the rigorous coordinates corresponding relationship between the original image pixel and the epipolar image pixel can be easily derived from the orientation data of satellite stereo imagery. Based on the Rational Polynomial Functions (RPF), G. Zhang et al. (2010) put forward a practical way of generating an epipolar image by the projection trajectory method and rebuilding its geometric model. In general, some deficiencies still exist in the above methods: (1) most of the methods are a kind of approximate method, and only the final resampled epipolar images are evaluated, lacking quantitative analysis in the whole process, especially on the epipolar lines; (2) present epipolar resampling methods are complicated for the epipolar geometry rectification, Level-2 images and DSM generation.

The remainder of this work is organized as, 1) In Section 2 the principle and foundation of the projection trajectory-based

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epipolarity model are introduced, and the properties of epipolar curve in the model is analyzed to address the problems mentioned above; 2) In Section 3, a novel epipolar rectification method is proposed based on the model leveraging the approximate conjugate property of epipolar line, followed by accuracy assessment and discussion on TH-1 dataset; 3) Section 4 concludes the work.

2. ANALYSIS OF PT-BASED EPIPOLARITY MODEL

2.1 The Projection Trajectory-based Epipolarity Model

Given the geometry of the frame perspective images can no longer be dealing with the linear pushbroom satellite images because differences existing in the imaging models, many researchers in aerospace photogrammetry community have made unremitting efforts and attempts to build the epipolarity model of linear pushbroom images. The most rigorous epipolarity model of the linear pushbroom image is the projection trajectory based on RPF.

As shown in figure 1, if all the points of the optical ray from the ground point Q through the left imaging centre $S_1(X_1, Y_1, Z_1)$ to the image point p on the left image, are projected to the right image I_2 through the right imaging center $S_2(X_2, Y_2, Z_2)$, the projection trajectory of these points will form a curve l on the right image. This curve l is termed an epipolar line of the point p . If q is the corresponding point of p , it is easy to know that q always locates on the epipolar curve l . This is the geometric definition of the projection trajectory-based epipolar line for the linear pushbroom satellite images.

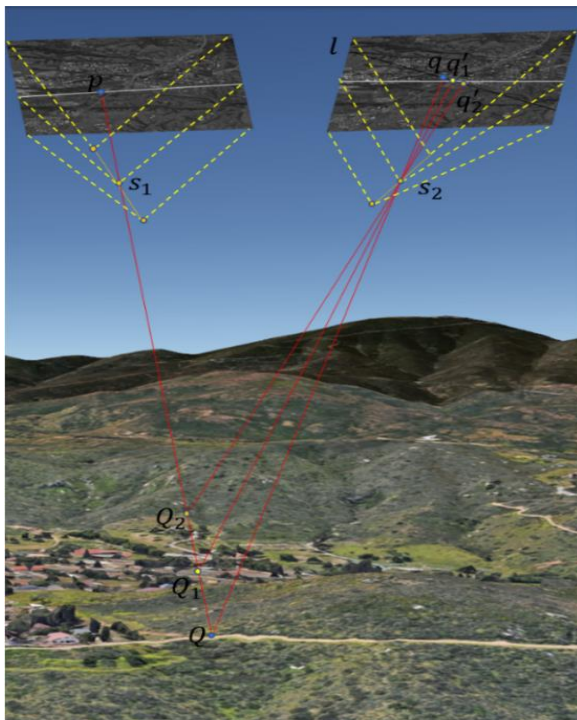


Figure 1. The projection trajectory-based epipolarity model.

The white line on the left and right images indicate the scan line when imaging, while the black line on the right image indicates the epipolar curve l of the point p on the left image.

It is easy to know that for the centre perspective camera, the epipolar line based on the projection trajectory method is linear. The results conclude that the epipolarity model based on the

projection trajectory method is completely consistent with the traditional model. Furthermore, the epipolarity model based on the projection trajectory method is more general. The existing studies have shown that the epipolar curve based on the projection trajectory method has the following properties:

- (1) The epipolar lines are hyperbola curves for the linear pushbroom satellite stereo images and can be handled as approximately straight lines on a small scale.
- (2) The corresponding points of point p in an epipolar line and its adjacent points in a certain range on an image are all located in the epipolar line of the point p . This conclusion is established at the local scale. According to this conclusion and conclusion (1), the search process of stereo image pairs matching can be constrained to one-dimension from two-dimension.
- (3) The corresponding epipolar line pair for the corresponding points exists. If two points are the corresponding points, then the corresponding epipolar lines are pairwise. Furthermore, all the corresponding points on the two epipolar lines are pairwise. The conclusion can be deduced by the conclusion (2) and the conjugate property of stereo image pairs.

2.2 Analysis of the PT-based Epipolarity Model

Previous analysis indicates that, on the local scale, the properties of the linear pushbroom satellite image epipolar line model are similar to the epipolar line of the traditional frame perspective image. So how about the linear approximation property in the image range? It needs further quantitative analysis to accurately grasp the epipolar line application properties and provide a theoretical basis for subsequent epipolar geometry rectification. Here two typical linear pushbroom commercial satellite images of IKONOS and GeoEye are selected as the experimental datasets for approximation linear property analysis of the epipolar curve.

2.2.1 Experimental dataset: Experimental dataset 1 consists of two IKONOS images covering a part area of Beijing, China, as is shown in Figure 2, which was after radiometric correction on the original image without any other geometry processing. The size of the image is 5057×8063 pixels, and the orientation parameters files are provided in standard RPC format.

Experimental dataset 2 is two GeoEye images, as is shown in Figure 3. The size of the image is 26928×15668 pixels, and the corresponding orientation parameters files are provided in standard RPC format.

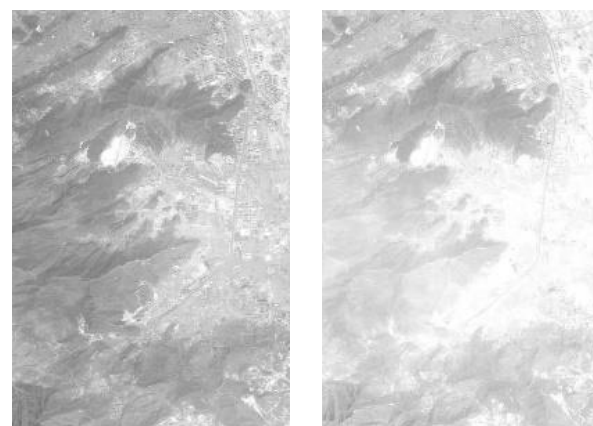
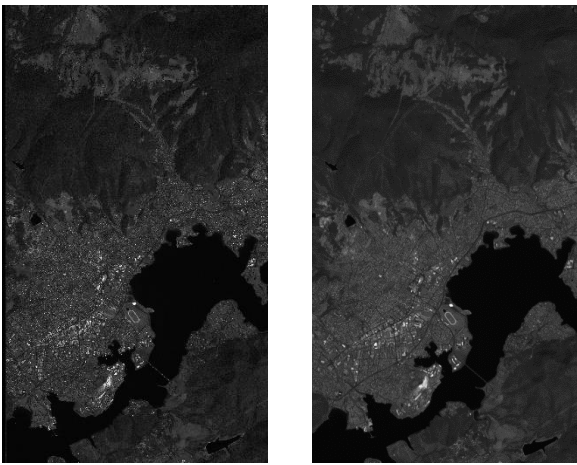


Figure 2. Overview of the IKONOS images dataset



Left image Right image
Figure 3. Overview of the GeoEye images dataset

In the analysis part on the PT-based epipolarity model of this work, a number of evenly distributed points are selected from the left image; indeed, left images are divided into 10×10 blocks, the centre point of each image block is selected as the test point, 100 test points are selected totally. The point distribution on the whole image is about ten lines multiply ten columns. According to image point coordinates of each test point on the left image and the RPC parameters of this image, straight space line through the projection centre and the image point is determined by the projection trajectory method. Then using the RPC parameters of the right image, the points on this space straight line are projected to the right image one by one. The projection trajectory of these points in the right image is the corresponding epipolar line. For the epipolar curve, an approximate line which is closest to this curve is determined by the least square method. Then the maximum distance between all points on the curve and the fitted straight line is computed. For the 100 test points, the above steps are executed gradually. With these steps, the curvature of the 100 lines, corresponding to these 100 test points, and the maximum distance between each curve and the fitted straight line are reported and analyzed.

2.2.2 Results and analysis on the IKONOS dataset: Table 1 describe the slopes of the straight lines, fitted from the epipolar curves, corresponding to the 100 evenly distributed points in the IKONOS dataset. Table 2 describes the maximum distance between all the 100 points on the curves and the straight lines fitted from these curves, which describes the linear fitting characteristic of the epipolar curve. It should be noted that the columns stand for the order of the 100 points in x coordinate direction, while the rows stand for the order of the 100 points in y coordinate direction. It is of importance to analyze the conjugate property of the PT-based epipolarity model. Thus, in order to verify the conjugate characteristic of the epipolar line, the same experimental method is adopted from the right image to the left image as shown in Table 3 and Table 4.

75.48788	75.49014	75.49058	75.48937	75.48817	75.48696	75.48576	75.48455	75.48335	75.48214
75.48814	75.49038	75.49035	75.48924	75.48812	75.48700	75.48588	75.48477	75.48365	75.48253
75.48841	75.49064	75.49010	75.48906	75.48802	75.48698	75.48593	75.48489	75.48384	75.48279
75.48869	75.49091	75.48992	75.48894	75.48795	75.48696	75.48596	75.48497	75.48397	75.48297
75.48903	75.49076	75.48981	75.48886	75.48790	75.48695	75.48599	75.48502	75.48406	75.48309
75.48930	75.49050	75.48957	75.48864	75.48770	75.48676	75.48582	75.48487	75.48392	75.48297
75.48957	75.49029	75.48936	75.48842	75.48748	75.48654	75.48559	75.48464	75.48369	75.48274
75.48983	75.49000	75.48904	75.48809	75.48712	75.48616	75.48519	75.48421	75.48323	75.48227
75.49008	75.48972	75.48873	75.48773	75.48673	75.48572	75.48471	75.48369	75.48267	75.48167
75.49031	75.48932	75.48827	75.48721	75.48614	75.48507	75.48400	75.48292	75.48183	75.48075

Table 1. Slopes of the fitting straight lines on the right image of the IKONOS dataset (degree)

Table 1 indicates that the curvatures of epipolar curves, corresponding to 100 evenly distributed points on the right image are somehow similar. The maximum value is 75.49091 degrees, and the minimum value is 75.48183 degrees, with a variation range being about 0.00908 degrees. The average slope value of straight lines fitted from curves is 75.487 degrees. If the average slope is regarded as the arranged orientation of the epipolar image, the maximal error exits at the edge of the image, which is about 0.415 pixels.

Table 2 reports that considering 100 points distributed evenly on the image, the fitting error is very little, and the maximum value is 0.15623 pixels.

0.00045	0.03811	0.07616	0.06810	0.06232	0.05710	0.05192	0.04678	0.04167	0.03661
0.00254	0.04969	0.07475	0.06929	0.06533	0.06141	0.05753	0.05368	0.04988	0.04611
0.00633	0.06270	0.07305	0.07054	0.06806	0.06562	0.06322	0.06086	0.05853	0.05624
0.01190	0.07718	0.07569	0.07455	0.07345	0.07239	0.07137	0.07039	0.06945	0.06855
0.01841	0.07724	0.07740	0.07759	0.07782	0.07809	0.07840	0.07874	0.07913	0.07955
0.02771	0.07889	0.08054	0.08222	0.08395	0.08571	0.08751	0.08935	0.09123	0.09314
0.03932	0.08349	0.08662	0.08980	0.09302	0.09627	0.09957	0.10290	0.10627	0.10247
0.05354	0.08750	0.09215	0.09684	0.10157	0.10634	0.11115	0.11599	0.12088	0.08580
0.07076	0.09459	0.10082	0.10710	0.11341	0.11976	0.12615	0.13258	0.13905	0.07147
0.09138	0.10102	0.10879	0.11660	0.12445	0.13233	0.14026	0.14823	0.15623	0.05629

Table 2. Maximum distances of the fitting straight lines on the right image of the IKONOS dataset (pixel)

Table 3 indicates that the curvatures of epipolar curves, corresponding to 100 evenly distributed points on the right image are also similar. The maximum value is 75.49093 degrees, and the minimum value is 75.48209 degrees, with a variation range being about 0.009 degrees. The average slope value of straight-line fitted from curves corresponding to 100 points is 75.487 degrees. If the average slope is regarded as the arranged orientation of the epipolar image, the maximal error exits at the edge of the image, which is about 0.415 pixels.

75.48784	75.48893	75.49022	75.48910	75.48797	75.48684	75.48571	75.48457	75.48342	75.48228
75.48818	75.49018	75.49014	75.48910	75.48805	75.48700	75.48595	75.48489	75.48383	75.48277
75.48858	75.49059	75.49010	75.48912	75.48813	75.48714	75.48615	75.48515	75.48415	75.48314
75.48893	75.49093	75.48999	75.48904	75.48810	75.48714	75.48619	75.48523	75.48427	75.48331
75.48932	75.49084	75.48991	75.48898	75.48805	75.48712	75.48618	75.48525	75.48430	75.48336
75.48966	75.49068	75.48975	75.48882	75.48788	75.48694	75.48601	75.48506	75.48412	75.48317
75.49004	75.49056	75.48961	75.48865	75.48770	75.48674	75.48578	75.48482	75.48385	75.48306
75.49041	75.49041	75.48942	75.48836	75.48736	75.48636	75.48535	75.48435	75.48334	75.48334
75.49071	75.49018	75.48913	75.48807	75.48701	75.48595	75.48488	75.48382	75.48275	75.48357
75.49106	75.49003	75.48890	75.48777	75.48664	75.48550	75.48437	75.48323	75.48209	75.48381

Table 3. Slopes of the fitting straight lines on the left image of the IKONOS dataset (degree)

Table 4 reports that considering 100 points distributed evenly on the left image of the IKONOS dataset, the fitting error is very little, and the maximum value is 0.135 pixel.

0.00037	0.03548	0.06924	0.06499	0.06071	0.05640	0.05207	0.04770	0.04332	0.03890
0.00213	0.04667	0.07035	0.06767	0.06497	0.06223	0.05947	0.05668	0.05386	0.05102
0.00491	0.05904	0.07917	0.08099	0.07777	0.06654	0.06257	0.06039	0.05626	0.05131
0.00945	0.07244	0.07202	0.07237	0.07288	0.07297	0.07324	0.07347	0.07368	0.07386
0.01475	0.07997	0.07249	0.07422	0.07592	0.07760	0.07925	0.08087	0.08246	0.08403
0.02221	0.07186	0.07510	0.07832	0.08150	0.08466	0.08779	0.09089	0.09397	0.09702
0.03029	0.07168	0.07620	0.08069	0.08515	0.08959	0.09400	0.09838	0.10274	0.10661
0.03980	0.07183	0.07756	0.08551	0.09141	0.09729	0.10314	0.10896	0.11475	0.08477
0.05222	0.07414	0.08127	0.08838	0.09546	0.10251	0.10953	0.11653	0.12349	0.09922
0.06512	0.07672	0.08510	0.09345	0.10178	0.11007	0.11834	0.12658	0.13480	0.05601

Table 4. Maximum distances of the fitting straight lines on the left image of the IKONOS dataset (pixel)

2.2.3 Results and analysis on the GeoEye dataset: Table 5 describe the slopes of the straight lines, fitted from the epipolar curves, corresponding to the 100 evenly distributed points in the GeoEye dataset. Table 6 describes the maximum distance between all the 100 points on the curves and the straight lines fitted from these curves, which describes the linear fitting characteristic of the epipolar curve. It is the same with the IKONOS dataset that the columns stand for the order of the 100 points in x coordinate direction, while the rows stand for the order of the 100 points in y coordinate direction. In order to verify the conjugate characteristic of the epipolar line, the same

experimental method is adopted from the right image to the left image. The results are shown in Table 7 and Table 8.

Table 5 reports that for GeoEye satellite image, the maximum orientation value of the epipolar line on the right image is 75.16487 degrees, and the minimum value is 74.14043 degrees, with a variation range being about 0.015 degrees. If the average slope (75.157 degrees) is regarded as the arranged orientation of the epipolar image, the maximal error exits at the edge of the image, which is about 1.96 pixels.

74.16522	74.16247	74.16028	74.15901	74.15775	74.15649	74.15524	74.15399	74.15274	74.15150
74.16487	74.16212	74.16018	74.15891	74.15765	74.15639	74.15514	74.15389	74.15265	74.15141
74.16446	74.16172	74.16008	74.15881	74.15755	74.15630	74.15504	74.15379	74.15255	74.15131
74.16411	74.16137	74.15998	74.15871	74.15745	74.15620	74.15494	74.15370	74.15245	74.15121
74.16370	74.16108	74.15982	74.15855	74.15729	74.15604	74.15479	74.15354	74.15229	74.15106
74.16336	74.16104	74.15978	74.15851	74.15725	74.15600	74.15475	74.15350	74.15226	74.15106
74.16295	74.16088	74.15961	74.15835	74.15709	74.15584	74.15459	74.15334	74.15210	74.15086
74.16254	74.16078	74.15951	74.15825	74.15699	74.15574	74.15449	74.15324	74.15200	74.15022
74.16219	74.16067	74.15941	74.15815	74.15689	74.15563	74.15438	74.15314	74.15190	74.14982
74.16178	74.16051	74.15924	74.15798	74.15672	74.15547	74.15422	74.15298	74.15174	74.14943

Table 5. Slopes of the fitting straight lines on the right image of the GeoEye dataset (degree)

0.00025	0.01595	0.03818	0.03811	0.03804	0.03797	0.03790	0.03783	0.03776	0.03769
0.00151	0.02224	0.03990	0.03983	0.03976	0.03968	0.03961	0.03954	0.03947	0.03939
0.00327	0.02802	0.03799	0.03792	0.03786	0.03779	0.03772	0.03765	0.03758	0.03751
0.00640	0.03618	0.03970	0.03963	0.03956	0.03949	0.03942	0.03935	0.03928	0.03921
0.00965	0.03968	0.03961	0.03954	0.03947	0.03941	0.03934	0.03927	0.03920	0.03912
0.01464	0.03957	0.03950	0.03944	0.03937	0.03930	0.03923	0.03916	0.03909	0.03926
0.01937	0.03948	0.03941	0.03935	0.03928	0.03921	0.03915	0.03908	0.03901	0.03903
0.02475	0.03759	0.03753	0.03746	0.03740	0.03734	0.03727	0.03721	0.03715	0.03707
0.03241	0.03928	0.03922	0.03915	0.03909	0.03902	0.03896	0.03889	0.03882	0.03876
0.03925	0.03919	0.03913	0.03906	0.03900	0.03893	0.03887	0.03880	0.03874	0.03833

Table 6. Maximum distances of the fitting straight lines on the right image of the GeoEye dataset (pixel)

Table 6 reports that considering 100 points distributed evenly on the right image of the GeoEye dataset, the fitting error is very little, and the maximum value is 0.039 pixel.

74.16546	74.16331	74.16148	74.16021	74.15894	74.15768	74.15642	74.15517	74.15392	74.15268
74.16498	74.16283	74.16114	74.15987	74.15861	74.15735	74.15609	74.15484	74.15359	74.15235
74.16447	74.16232	74.16077	74.15950	74.15824	74.15698	74.15572	74.15447	74.15323	74.15198
74.16399	74.16184	74.16046	74.15920	74.15794	74.15668	74.15543	74.15418	74.15293	74.15169
74.16348	74.16136	74.16009	74.15883	74.15757	74.15631	74.15506	74.15381	74.15257	74.15133
74.16297	74.16103	74.15976	74.15849	74.15723	74.15598	74.15473	74.15348	74.15224	74.15100
74.16249	74.16069	74.15942	74.15816	74.15690	74.15565	74.15440	74.15315	74.15191	74.15057
74.16198	74.16036	74.15909	74.15783	74.15657	74.15532	74.15407	74.15282	74.15158	74.15010
74.16150	74.16002	74.15876	74.15750	74.15624	74.15499	74.15374	74.15249	74.15125	74.14960
74.16099	74.15966	74.15839	74.15713	74.15587	74.15462	74.15338	74.15213	74.15089	74.14910

Table 7. Slopes of the fitting straight lines on the left image of the GeoEye dataset (degree)

Table 7 reports that for GeoEye satellite image, The maximum orientation value of the epipolar line on the left image is 74.16546 degree and the minimum value is 75.14910 degrees, with a variation range being about 0.016 degrees.

0.00005	0.00842	0.02160	0.02157	0.02154	0.02151	0.02148	0.02145	0.02142	0.02140
0.00051	0.01191	0.02265	0.02262	0.02259	0.02256	0.02253	0.02250	0.02247	0.02244
0.00133	0.01513	0.02267	0.02264	0.02261	0.02258	0.02255	0.02252	0.02249	0.02246
0.00290	0.01971	0.02270	0.02267	0.02264	0.02261	0.02258	0.02255	0.02252	0.02249
0.00459	0.02276	0.02273	0.02270	0.02267	0.02264	0.02261	0.02258	0.02255	0.02252
0.00668	0.02176	0.02173	0.02170	0.02167	0.02164	0.02161	0.02158	0.02155	0.02152
0.00984	0.02283	0.02280	0.02277	0.02274	0.02271	0.02268	0.02265	0.02262	0.02259
0.01280	0.02183	0.02180	0.02177	0.02174	0.02171	0.02169	0.02166	0.02163	0.02160
0.01707	0.02291	0.02288	0.02285	0.02282	0.02279	0.02276	0.02273	0.02270	0.02267
0.02093	0.02295	0.02292	0.02289	0.02286	0.02283	0.02280	0.02277	0.02274	0.02271

Table 8. Maximum distances of the fitting straight lines on the left image of the GeoEye dataset (pixel)

Table 8 reports that considering 100 points distributed evenly on the left image of the GeoEye dataset, the fitting error is very little, and the maximum value is 0.023 pixel.

Comparing with the results on the IKONOS dataset, the epipolar line slope variation range of GeoEye image is larger. That is to say, the parallelism property of the epipolar line is weak. However, the approximate linear property of all epipolar lines is similar to the IKONOS dataset. For the IKONOS image, when the precision range is in the sub-pixel level, epipolar lines on the

image can be considered parallel to each other. However, for the GeoEye dataset, epipolar lines are not parallel. This is partially due to the size of the experimental dataset changes greatly, while the size of the IKONOS image used in the experiment is only 5057×8063, and GeoEye is 26928×15668. Experiment results on the IKONOS dataset report that in the whole image range, the PT-based epipolarity model has good approximate linear property and approximate parallel relationship between each epipolar line. Based on the approximate parallel property, the rectification of the epipolar geometry becomes intuitive. Rotated directly by the epipolar line slope angle, the epipolar image can be resampled with very little vertical parallaxes.

It can be concluded from the above analysis: the projection trajectory-based epipolarity model has distinct different parallel properties on linear pushbroom satellite images with different sizes. Nevertheless, in a local range in the image, the approximate linear property is satisfied for the sub-pixel level requirement. Consequently, this projection trajectory-based epipolar curve can be handled as a straight line in the local-scale image.

3. EPIPOLAR IMAGE RESAMPLING

It is of importance to generate the epipolar images, which is one of the critical constraints for stereo epipolar images matching in the practical work of surveying and mapping. Traditional methods of generating epipolar images can be basically divided into two categories: one is based on geometric correction of digital images; The second is based on the coplanar condition. The method based on digital image geometric correction is essentially a digital correction, projecting the epipolar line of the oblique image to the ortho-ready image plane to obtain corresponding epipolar lines of ortho-ready images. The coplanar condition-based method can directly determine the direction of the epipolar line on the oblique image, then determine the location of the epipolar line on the oblique image according to the coordinates of one epipolar line's start point and the direction of epipolar line.

Referring to the epipolar image generation method based on coplane condition, we proposed a method to rectify the epipolar geometry utilizing the properties of the projection trajectory-based epipolarity model as discussed in the aforementioned sections. Referring to the method of traditional frame perspective remote sensing images, which samples the epipolar image from the oblique image directly, the epipolar image of the whole area is acquired through processing line by line. According to the epipolar model assessed above, when the point in the left image is known, the optical ray determined by left image point p and the projection centre S_1 of the left image is projected on the right image and formed a curve l ; then, a straight line is fitted to approximate the curve, and the straight line is decided as an epipolar line on the right image. In the same way, the optical ray determined by right image point q and the projection centre S_2 of the right image is projected on the left image to form a curve l' . A straight line is fitted to approximate the curve, and the line is epipolar line l' corresponding to l . Finally, the equation parameters of l and l' are used to resample epipolar lines in the original image directly.

In addition, for the generated epipolar images, the epipolar image coordinates can be converted from the original image coordinates by the parameters of the two straight lines. It is critical information that should be recorded and thus can be applied for computing digital surface models on the translation stage from the original image pairs.

3.1 The Experimental Dataset

A Tianhui-1 (TH-1) satellite dataset is selected in this section, as shown in Figure 4. This dataset contains three scenes of images and the corresponding RPC files. TH-1 is China's first transmissive optical stereo mapping satellite. The satellite was launched on August 24, 2010, which carries on three kinds of loads including three linear array stereo mapping cameras, a high-resolution camera and four bands multi-spectral camera. The resolution of three linear-array panchromatic images is better than 5 meters (including forward, nadir and back). The resolution of the multi-spectral image is better than 10 meters. The resolution of the high-resolution image is better than 2 meters.

In this experiment, one scene of TH-1 three linear array image in Baoji, Shaanxi region in July 2011 is chosen as is shown in Table 9 below. The cover area of this dataset is 3600 square kilometres (60 km×60 km). There is no cloud coverage in the images. The elevation difference is moderate, which has both mountain and urban areas.

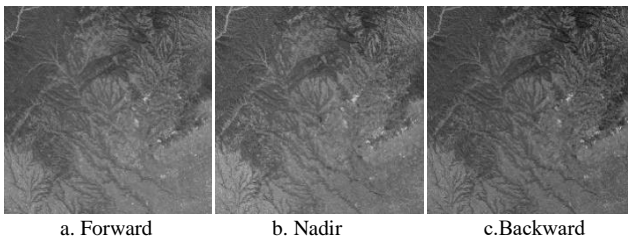


Figure 4. Overview of the TH-1 Satellite Images

3.2 Methodology for Epipolar Image Resampling

Taking an example of the TH-1 satellite stereo image pair generation with forward and nadir, the detailed steps are below:

- (1) Starting from the first column in the forward image, the middle image point $m(0, H/2)$ in the column direction is selected. Based on the PT-based epipolarity model, the line determined by point m and the projection centre S of the left image is projected on the right image. Considering the computation, in this paper only two object space points Q_1 and Q_2 of this line on elevation h_{max} and h_{min} are projected on the right image to determine the two image points q_1 and q_2 .
- (2) Determine whether the point q_1 and q_2 are in the range of the right image. If not, return to (1) and continue to start from the next column. According to these two points, a straight line l' is determined and the line $a'x + b'y + c = 0$ is recorded.
- (3) Determine the midpoint \hat{q} between q_1 and q_2 on the right image. According to the PT-based epipolarity model, the optical ray determined by the point q and the projection centre S' of the right image is projected on the left image. Considering the computation simpleness, in this work only two object space points Q_3 and Q_4 of this optical ray on elevation h_{max} and h_{min} are projected on the left image to determine the two image points q_3 and q_4 .
- (4) Determine whether the point q_3 and q_4 are in the range of the left image or not. If not, return to (3) and continue to start from the next column. According to these two points, a straight line l is determined and recorded: $ax + by + c = 0$.

Using the straight-line l on the left image, carry out resample along the straight line on the left image. Given a series of y_i coordinates varied from 0 to H (the height of the image),

according to the straight-line equation, the corresponding series of x_i coordinates are computed. According to the series of (x_i, y_i) , the corresponding series of pixel grey values are determined from the original image by bilinear interpolation. This series of grey values is the first line on the left epipolar image;

Also, the first line on the right epipolar image can be determined according to l' . Return to the first step and repeat the steps until the last line on the epipolar image is determined.

Record the parameters of every straight line equations on the left image. The group number of equation parameters are the same as the number of epipolar lines. Save the straight line parameters into a text file. This parameter file can be used to realize the transformation from the epipolar image coordinates to the original image coordinates. Carry out the same process on the right image to form a parameter file. The corresponding relationship of the epipolar image and the original image is:

$$\begin{cases} y = \text{epiwidth} - x' - 1 \\ x = -\frac{b_i}{a_i}y - \frac{c_i}{a_i}, i = 0, \dots, n - 1; \\ x' = 0, \dots, \text{height} - 1 \\ y' = i \end{cases}$$

Where x and y are the pixel coordinates on the original image, which are the number of columns and rows respectively; And x' and y' are the pixel coordinates on the epipolar image, which are the number of columns and rows respectively; height represents the rows number of the original image; n is the number of epipolar equations; i is the number of straight lines; a_i , b_i , c_i indicate the parameters of the line equations.

3.3 Result Assessment

The epipolar images for TH-1satellite images are generated by the method proposed in Section 3.1, including the forward image, the nadir image, and the backward image. As shown in Figure 5.

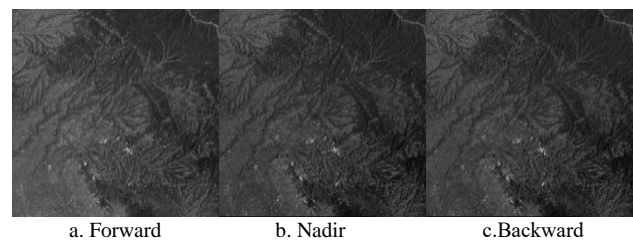


Figure 5. Computed Epipolar Images

In order to verify the vertical parallax of the epipolar image, firstly the stereo mapping module of VirtuoZo was adopted to select 31 pairs of evenly distributed corresponding in the epipolar stereo image pair. Then the coordinates and its vertical parallax of these corresponding image points in the epipolar stereo image pair are computed, and the results are shown in Table 9.

No.	x_1	y_1	x_2	y_2	$\ y_2 - y_1\ $
1	1938.060	424.298	1933.560	424.798	0.500
2	2046.670	336.671	2043.840	336.545	0.126
3	2751.060	421.832	2753.560	422.332	0.500

4	3230.800	407.298	3238.210	407.430	0.132
5	5615.880	93.154	5651.220	93.530	0.375
6	7125.080	476.220	7141.990	475.399	0.321
7	6163.620	485.031	6192.150	485.291	0.260
8	8333.200	387.814	8344.730	387.898	0.084
9	9879.620	463.114	9880.800	463.315	0.201
10	10076.300	85.529	10071.200	85.854	0.325
11	4799.670	568.886	4824.130	569.157	0.271
12	5108.540	928.497	5123.040	928.997	0.500
13	5747.120	525.344	5786.310	525.627	0.283
14	10218.500	1054.620	10226.600	1054.530	0.090
15	3951.420	2020.160	3960.610	2020.370	0.210
16	4375.430	2352.690	4396.200	2353.280	0.590
17	5966.150	2172.420	5996.160	2172.570	0.150
18	6373.360	2013.730	6375.130	2014.000	0.270
19	6830.020	2087.540	6852.540	2087.680	0.140
20	7959.540	2085.640	7983.410	2085.860	0.220
21	10323.200	8150.030	10318.800	8150.020	0.010
22	10846.200	8357.120	10887.000	8356.970	0.150
23	7487.890	8727.050	7469.060	8727.410	0.360
24	8401.910	8864.590	8402.980	8864.250	0.340
25	10410.900	8721.000	10424.400	8720.830	0.170
26	10563.800	8744.680	10600.200	8744.550	0.130
27	11105.600	8931.260	11125.300	8931.180	0.080
28	316.417	9333.660	281.288	9333.310	0.350
29	7763.480	11240.700	7781.980	11240.700	0.500
30	10481.100	11039.400	10478.100	11039.200	0.200
31	11622.100	11226.400	11642.100	11226.500	0.100

Table 9. The Vertical Parallax evaluation

x_1 is the column value of the corresponding point on the forward epipolar image, and y_1 is the row value; x_2 is the column value of the corresponding point on the back epipolar image, while y_2 is the row value. The vertical parallax ($\text{abs}(y_2 - y_1)$) of the epipolar image is computed to describe the precise of epipolar images, especially the left and right edge points. Table 1 indicates that the vertical parallax of corresponding image points involved in the inspection can achieve accuracy at sub-pixel and root mean square error can achieve accuracy up to 0.590 pixels. It shows that this method can generate a quasi rigorous satellite epipolar image.

4. CONCLUSION

The contribution of this work is two folds. On the one hand, we explored the application of the rational function model in the projection trajectory-based epipolarity model for linear pushbroom satellite images. The projection trajectory-based epipolarity model is analyzed quantitatively using IKONOS and

GeoEye datasets. The approximate linear and parallelism property of the epipolar curve in the image is analyzed.

On the other hand, for the good approximate linear property of epipolar images, referring to the epipolar line sampling method of traditional frame perspective images, a method of epipolar image generation is proposed, which is similar to the method based on coplanar conditions. Based on the projection trajectory-based epipolarity model, treating the epipolar line as the approximate straight line, this method established a corresponding rigid relationship of coordinates between the original image and epipolar image by the orientation parameters of stereo images, and then carried out the epipolar image resample directly on the oblique image, and finally generated quasi rigorous epipolar image pairs with the vertical parallaxes in sub-pixel level.

The proposed method is practical and straightforward, which is performed on the TH-1 satellites images in this work. At present, the proposed method has been successfully applied to the follow-up stereo mapping and automatic DSM generation software for the TH-1 satellite images.

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