Visualization of Space Occupancy Uncertainty in a 3D Voxel-based Urban Model

Olga Shkedova[1], Monika Sester[1]

[1]Institute of Cartography and Geoinformatics, Leibniz University, Hannover, Germany - Olga.Shkedova@ikg.uni-hannover.de, Monika.Sester@ikg.uni-hannover.de

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Abstract

In a 3D voxel-based digital urban model - or digital urban twin, the semantics of each cell is typically visualized using RGB values to give a realistic impression. However, due to measurement uncertainties or incompleteness, some voxel occupancy might be less likely than others. This paper suggests computing an occupancy grid map using an inverse sensor model and embodying the obtained occupancy probabilities into the urban digital twin.
This article then presents visualization of additional information, in this case the uncertainty in space occupancy, in a 3D voxel-based urban model. To this end, a concept is proposed, in which static visual variables are complemented by dynamics. It suggests to utilize the visual variables transparency, size, and color, supplemented by animation. The main idea is to communicate occupancy uncertainty information and enhance its perception via visual variables while visually inspecting a digital twin in an animation mode, observing the occupancy uncertainty without losing the perception of the urban environment.

1. Introduction

Intensive improvement in technologies, methodologies, and instruments for data collection and 3D city modeling has created various opportunities for applications addressing challenges, including those related to urban planning, infrastructure organization, disaster management, and beyond. In this context, the concept of the smart city became popular in the 2010s, which involves administration and citizens working together with new technologies to make the city more efficient, smarter, more sustainable and safer, and cities were equipped with sensors (Arroub et al., 2016).

Modern smart cities are functioning by integrating data from different sensors and semantic, geospatial properties of city objects into the 3D city model, creating a precise digital replica, termed an urban digital twin (Juarez et al., 2021; Tomko and Winter, 2019; Castelli et al., 2019). The continuous data exchange ensures that the model within the digital twin closely reflects the real city and its systems, providing a highly accurate representation. This interactivity has the potential to offer substantial advantages in the management and operation of urban infrastructure (Ferré-Bigorra et al., 2022).

Nowadays, several urban digital twins are operating. For instance, German 3D Spatial Base Data provides 3D city models with the Level of Detail 1 (LoD1) and 2 (LoD2) (Gruber, 2020). The digital twin of Vienna, Austria, focuses on the geodetic and geometric aspects of semantic geo-objects (Lehner and Dorffner, 2020). The Helsinki digital twin, Finland, comprises 3D city models, the utility of which can be expanded to assess solar energy potential, conduct noise emission simulations, or test the impact of flooding, allowing for linkage with supplementary data and information such as census data, socio-economic data, energy consumption data, maintenance management data, and more (Ruohomäki et al., 2018).

The majority of modern urban digital environments consist of 3D city models realized within the Building Information Modelling (BIM) or CityGML framework (Gobeawan et al., 2018; Schrotter and Hürzeler, 2020; Yang and Kim, 2021; Dembski et al., 2020). Although these types of models can be highly accurate, keeping them consistently updated to reflect real-time changes in the city poses a considerable challenge (Tang et al., 2018).

Data reliability and correct alignment of virtual urban indoor and outdoor environments, also under real-time conditions are crucial for services such as emergency operations, navigation, autonomous vehicle route planning, and smart space management. Therefore, a voxel-based representation method for urban digital twins has been recently proposed by Mortazavi et al. (2023). The fundamental idea behind this project is to capture high-resolution 3D features of the urban environment using diverse sensor systems and partitioning the data into billions of voxels that can be systematically updated, potentially in a decentralized way (Mortazavi et al., 2023). Additionally, the developments in data transmission technologies, such as 5G, promise real-time update applications for digital twins in the near future, leading to dynamic urban information. In the voxel representation, the necessary supplementary information is linked not to the 3D model objects, but to each voxel, thereby making an urban model more flexible for real-time changes due to the possibility of direct voxel update without a preprocessing step. Moreover, voxel-based models are actively employed in several areas and disciplines, e.g. to define the unexplored space for autonomous vehicles and smart space management, robotic operations, navigation, and route planning, computer graphics, rendering techniques, classification, localization and 3D reconstruction tasks that can significantly extend the range of urban digital twin applications in the future (Agus et al., 2010; Heajung et al., 2023; Hübner et al., 2022; Deng et al., 2024; Mortazavi et al., 2023).

Aside from 3D modeling, visualization is another crucial component of urban digital representation. Visualization plays a substantial role in ensuring the digital twin’s realism, interactivity, and scalability (Martella et al., 2023). The visualization of the urban 3D model should not only display the model itself but also be tailored to meet the specific requirements of the applic-

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ation, filtering out unnecessary details while emphasizing key information (Somanath et al., 2023). Therefore, the proposed 3D voxel-based urban environment requires adequate semantic information visualization and data integration methods for potential use cases. As the model will be continuously updated, corresponding visualization of the dynamic urban environment according to the real-time changes is required as well.

In the voxel representation, each cell contains information about its status. Typically, this relates to the objects and their semantics, e.g. a building and its usage. In the context of urban space management, it is also very important to know, if a space is occupied or not. This is e.g. necessary, to assign a space to be used for a farmer’s market on Thursday and Saturday afternoons; similarly, spaces can be allocated for parking of courier vehicles at their delivery times. Another usage of the occupancy of space is to determine if a vehicle can pass through a narrow passage.

Thus, our research is directed toward exploring the integration and visualization of such additional data into the 3D voxel-based urban model. We aim to investigate how such additional information can be effectively incorporated and perceived within the urban digital twin. This paper presents a design concept and initial results for the visualization. It is proposed to communicate the occupancy status of the space and its uncertainty using static visual variables such as transparency, color, and size. These variables are supplemented by animation, which assists users in observing the urban environment and the occupancy space uncertainty at the same time.

2. State of the Art of Visualization and Uncertainty Visualization

2.1 Visual Variables

A voxel-based urban digital twin embodies geodata collected by sensors and can be readily interpreted as a 3D georeferenced map. Therefore, in our research, we focus on the visualization aspects utilized in geospatial information representation techniques. Geovisualization provides intuitive and practical methods for abstract geographic information understanding and communication through graphical symbols. Acting as an intermediary between users and geospatial data, a reliable visualization system should help users efficiently and accurately identify target symbols within a visual context (Swienty et al., 2007).

Bertin’s visual variable theory posits that graphical symbols can be represented by seven fundamental visual variables: position, size, shape, color, orientation, and texture (Bertin, 1987). Further, the research of Koussoulakou and Kraak (1992) showed that animation assists in comprehending message contents more effectively than employing traditional static maps. Consequently, six dynamic visual variables were proposed by DiBiase et al. (1992) and MacEachren (2004), denoted as moment, duration, frequency, order, rate of change, and synchronization. Since then, with the advancement of technologies, these variables have been expanded and enhanced across various platforms, including immersive maps, augmented reality, virtual reality systems, and gamification engines. For instance, Zhang et al. (2023) proposed extending visual variables such as color and size using natural material color, illuminating material color, linear and angular sizes. Additionally, they conducted research involving AR geovisualization user experiments, where dynamic variables like vibration and flicker were found to offer the highest guidance. Mao et al. (2020) mixed the HSL color space visual variable and transparency to visualize the heat level and energy loss respectively, for an energy simulation online framework based on a 3D city. Gautier et al. (2020) used a 3D point cloud with varying point sizes and densities, along with animation, to display the different temperature degrees in a 3D city model.

2.2 Visualization of Geospatial Information Uncertainty

In recent decades, scholars in geographic information science have designed and assessed visualizations of uncertainty across various contexts (McKenzie et al., 2015). MacEachren et al. (2005) describe in detail the fundamental aspects of uncertainty visual representation, considering combinations of visual variables, glyphs, dynamic changes in data colors, uncertainty in 3D bars, and other methods. Brodlie et al. (2012) compiled a catalogue of visualization techniques, detailing the research efforts dedicated to expanding each method’s capability to address uncertainty. In their book, the authors acknowledge that historically, the geovisualization community was perhaps the first to realize the importance of uncertainty, providing examples such as the visualization of Digital Elevation Models (DEMs) uncertainty using animation, counter uncertainty visualization for oceanography maps, texture-based approaches for representing surface velocity effects in ocean maps, among others. Kinkeldey et al. (2014) conducted an extensive review of user studies on geospatial uncertainty visualization. Their work concluded that the majority of the studies are based on the manipulation of existing map content to represent uncertainty by utilizing color hue, color value, color saturation, and transparency.

Furthermore, visual variables were extensively extended for the 3D uncertainty visualization techniques including opacity, pseudo-coloring, fuzziness, side-by-side comparison, and various animations. For example, Huang et al. (2019) introduced an approach combining realistic visualization of a natural forest environment with uncertainty information. Their method, termed the ”slide-and-show” technique, enables users to explore the uncertainty of the model by adjusting a slider interface, which dynamically represents the corresponding density of trees. The study of Dübel et al. (2017) introduced a range of design options enabling the generation of prioritizing visual representations of 3D terrain models and uncertainty information. These options include the utilization of a continuous color scale, color-coded triangular glyphs and circles integrated into the terrain, transparency adjustments, phong illumination, ambient occlusion, ambient aperture lighting, realistic surface texture, and other techniques. O’Banion et al. (2019) presented an interactive visualization of 3D coordinate uncertainties of terrestrial laser-scanning point clouds using OpenGL shader language for uncertainty and point cloud colors blending.

3. Occupancy Map Modeling

Occupancy information is an important knowledge that is widely used in the mobile robotics field and utilized for localization, path planning and space management applications. In the last decades, occupancy grid maps have become a prevailing approach for modeling environments. These maps serve as spatial representations of the environment, employing a grid structure where each cell denotes qualitative information regarding its state, which indicates the certainty of occupancy based on sensor measurements (Thrun, 2003; Dia et al., 2018). The conventional approach for constructing occupancy grid maps often involves the method proposed by Elfes (1987), which leverages inverse sensor models. In this method, the mapping problem is approached inversely to the generation of sensor data as follows:

$$p(M|z_{1:T}, x_{1:T}), \tag{1}$$

where M = entire map
$z_{1:T}$ = complete set of measurements
$x_{1:T}$ = corresponding poses

To simplify the mapping problem, two assumptions are made. The first assumption is that cells are conditionally independent given measurements and the trajectory of the robot. The second assumption, known as the static world assumption, considers a measurement at time t to be conditionally independent of previous measurements given the map knowledge. Considering the assumptions and Bayes' rule, the log odds l of the probability of occupancy for all grid cells i, including updates for the cells within the sensor range of the measurement z_t, can be computed as follows:

$$l_i^t = \log \frac{p(m_i|z_t, x_t)}{1 - p(m_i|z_t, x_t)} - \log \frac{p(m_i)}{1 - p(m_i)} + l_i^{t-1}, \tag{2}$$

where i = prior of occupancy of the cell
$p(m_i)$ = probability term
$l_i^t = log \frac{p(m_i|z_{1:t}, x_{1:t})}{1-p(m_i|z_{1:t}, x_{1:t})}$
$l_i^{t-1} = log \frac{p(m_i)}{1-p(m_i)}$

The computation of the log-odds occupancy representation for cells within the sensor measurement coverage cone is straightforward. As a result, the desired occupancy probability of these cells can be obtained as follows:

$$p(m_i|z_{1:t}, x_{1:t}) = 1 - \frac{1}{1 + e^{l_i^t}} \tag{3}$$

An occupancy value close to zero indicates the high probability that the corresponding area is free from obstacles. Hence, a value close to one signifies that the area is occupied. The occupancy value of 0.5 indicates an equal likelihood of occupancy or vacancy and can be interpreted as an unknown space.

4. Concept for Visualization of Occupied Voxels and their Uncertainty

The problem when conveying the occupancy status of voxels is that the information should not be confounded with the (semantic) object information. Thus a mix of static variable and dynamic animations is proposed.

To convey the occupancy state of the environment to users, the following static visual variables are selected: transparency, size, and color (hue). The idea is to communicate the certainty of an occupied space to the user, whereby unsafe spaces should be perceived as less prominent by applying transparency and size visual variables, while the opposite effect of emphasizing and signaling the uncertainty of occupied space is obtained with color visual variable. This is achieved by the following means and effects:

1. Transparency: uncertain spaces have low opacity
2. Size: uncertain spaces have small voxel size
3. Color: uncertain spaces are emphasized with yellow, orange, and red colors (whereas certain ones are given in green)

The choice of visual variable type can be applied depending on the specific use case. Therefore, the transparency variable aims to gradually decrease the opacity level as the probability value of voxels decreases. Similarly, the size variable decreases the size of the voxel as the probability value decreases. The color variable is employed to inform users about the certainty of voxel occupancy, displaying occupied voxels with high probability in green, transitioning through yellow, orange, and finally red as the probability decreases. Due to the substantial volume of data, probability thresholds are categorized into four levels: very high probability, exceeding 0.9; high probability, ranging from 0.7 to 0.9; low probability, between 0.6 and 0.7; and very low probability, higher than 0.5 but lower than 0.6. Corresponding values for each threshold, opacity levels for the transparency variable, scale values for the size variable, and colors are detailed in Table 1.

Probability	Opacity	Scale	Color (Hex)
0.9<	1	1	008000
0.7-0.9	0.5	0.5	FFFF00
0.6-0.7	0.25	0.25	FFA500
0.5 <	0.1	0.1	FF0000

Table 1. The values for opacity levels of the transparency variable, scale values of the size variable, and colors specified according to the probability thresholds.

The primary concept for representing occupancy information in the urban environment is to ensure that users can comprehend the occupancy situation without losing the perception of the urban environment - and not confounding the occupancy status with the object information. For this reason, we augment the static visual variables with dynamic variables such as duration. In this way, users can observe the urban environment while the visual variables are animated according to the following function:

$$value + (1 - value) \cdot (\sin(animationtime/duration \cdot \pi)), \tag{4}$$

where $value$ = opacity or scale value (Table 1)
$animation\ time$ = animation time span
$duration$ = period of a visual variable
sin = periodic function of visual changes

The animation time refers to the time from the start of the animation, while the duration is an adjustable time parameter that determines the appearance time of the visual variable for the user. The *sin* function provides an option to gradually decrease opacity level and voxel size until reaching the threshold value over the duration time and then repeat the animation. In the case of the color visual variable, the voxels are changing the initial RGB colors to the occupancy status colors (see Table 1) over the defined duration time, using an overlaying technique.

5. Implementation of the Concept

5.1 Data

The dataset for voxel-based urban model generation comprises a sample point cloud and trajectory information, which includes time, location, and rotation of the laser scanner obtained by the Riegl VMX-250 system (MMS) mobile mapping system in the city of Hannover. The point cloud and the trajectory are represented in Figure 1.

Figure 1. The point cloud measured by the Mobile Mapping System, with the trajectory of the sensor movement, represented by the magenta line.

5.2 Occupancy Grid Computation

The 3D occupancy grid is generated based on the point cloud dimensions with a resolution of 10 cm. Further, the inverse sensor model, described in Section 3 is utilized to calculate the log odds values for each voxel in a grid cell based on the sensor data according to 2. In the next step, the log odds values are converted to the probabilities following 3. Figure 2 represents the obtained 3D occupancy grid.

5.3 Voxel-based Urban Model Visualization

Due to the utilization of point cloud coordinates in the grid generation process, the cell indices are interpreted as geographical coordinates of 3D voxels. RGB color information is derived by averaging the color values of points within each voxel, enabling the representation of the voxel-based urban environment. The probability values of the grid are saved for all voxels as attributive information.

For the urban model visualization, the voxels of the grid resolution size are generated using the cross-browser JavaScript library ”Three.js” and positioned according to the geographical location. To accommodate the rendering scene, the voxel coordinates are adjusted through scaling and a 90-degree rotation around the X-axis. Furthermore, the urban environment is represented in a web browser using a Python HTTP local server.

Figure 3 illustrates the urban digital twin fragment visualization before the visual variables animation.

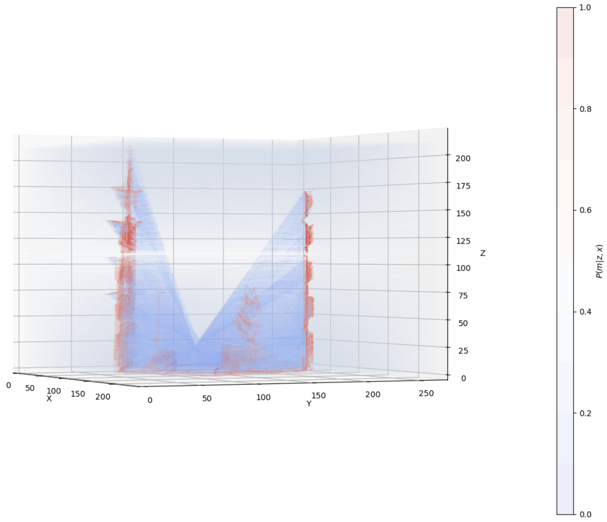


Figure 2. The occupancy map of the urban environment, defined by the probability values. The red color indicates the occupied cells, grey color represents unknown space and blue corresponds to free areas.

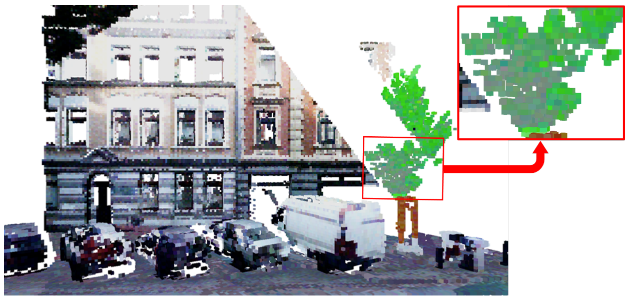
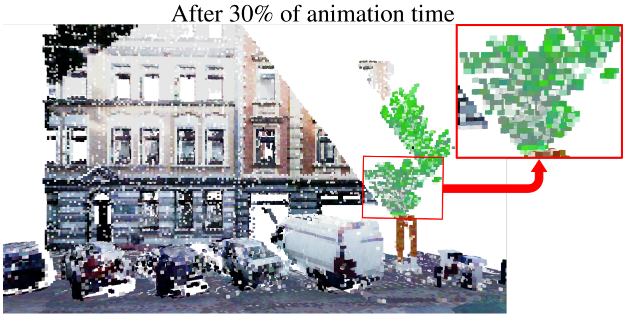


Figure 3. The fragment of the urban environment (Hannover) before the visual variables animation.

Figures 4 to 6 show the performance of the visual concepts introduced in Section 4: The results with the visual variables transparency, size, and color are shown after 30% of animation time and after 50% of animation time. It should be noted that small videos have been included as supplementary materials for this article to better convey the animation effect.

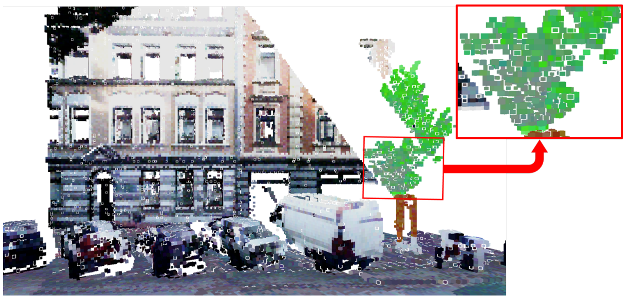
In Figure 4 it can be observed that as animation time progresses, some voxels become less perceptible while others almost disappear. For example, the green leaves near the tree trunk are clearly visible in Figure 3, but after 30% of the animation time, the leaves begin to become transparent, and at 50%, some leaves are no longer visible. Additionally, it can be noticed that the changing transparency of voxels over time during animation introduces noise to the overall visual impression. Figure 5 illustrates how the voxels, which appeared as a complete area in Figure 3, become more and more sparse, when applying size as a visual variable. It can be noticed how the size of the voxels representing the green leaves near the tree trunk changes with the animation’s duration. Likewise, the road pavement around the cars is depicted entirely in Figure 3, whereas in Figure 5, we observe that as the animation progresses, the road pavement becomes increasingly incomplete. Finally, Figure 6 illustrates the effect of the visual variable color and shows how the initial colors of voxels gradually turn to green, yellow, orange, and red colors, communicating to the user the uncertainty level.

After 30% of animation time

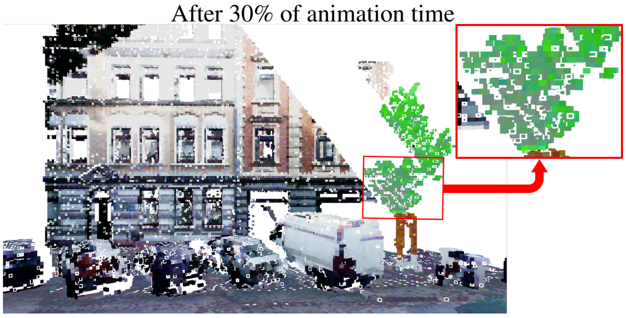


After 50% of animation time

Figure 4. Performance of visual variable transparency according to the occupancy probability states.



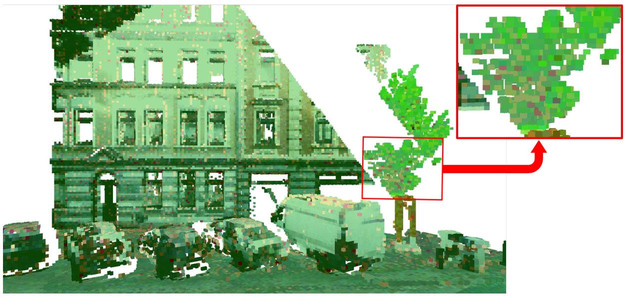
After 30% of animation time



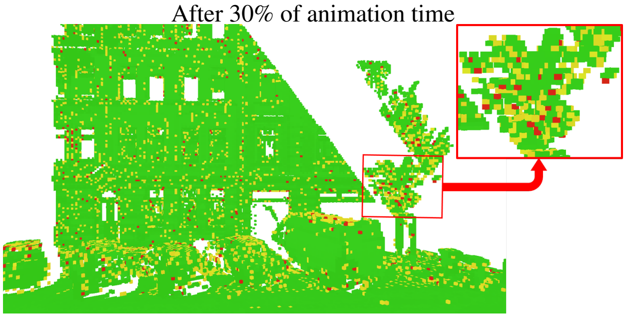
After 50% of animation time

Figure 5. Performance of visual variable size according to the occupancy probability states.

A first assessment of the effects shows that the visualization using the color metaphor clearly conveys the intended information, namely the fact that in most parts of the scene, the information is certain (shown in green), and only in areas at the boundaries of the objects the information is less sure (as shown in red). This clear communication comes at the expense of visualizing the semantic information of the environment, which, due to animation, allows for the analysis of occupancy uncertainty. The changes in transparency and size allow the observer to clearly perceive the space with the highest probability of occupancy. This voxel space is always preserved in size and opacity, while voxels with low occupancy probability become less perceptible



After 30% of animation time



After 50% of animation time

Figure 6. Performance of visual variable color according to the occupancy probability states.

during the animation due to a decrease in opacity and size, eventually almost disappearing from the scene in cases with lowest probability. The benefit of such a communication is to allow the identification of data in terms of occupancy uncertainty, without disrupting the perception of the urban environment with real-world RGB colors.

6. Conclusion and Outlook

The article outlines the initial results of an approach for integrating semantic information, such as occupancy uncertainty and visualizing the occupancy-aware voxel-based urban environment. By leveraging point cloud and trajectory sensor data, the inverse sensor model for computing occupancy grids is implemented, and the resulting occupancy probabilities are integrated into the urban model. To effectively visualize an occupied environment, the utilization of static visual variables, such as transparency, size, and color, along with animation, is proposed to emphasize the occupancy status to the user while maintaining the perception of the urban digital twin.

The next step is to evaluate the efficiency and effectiveness of the proposed visual variables in terms of communicating appropriate occupancy awareness to users in a large-scale user study. Additionally, the significance of preserving the exact representation of the urban environment for users during secondary information visualization is to be investigated. Moreover, future research aims to explore additional static and dynamic visual variables, such as texture, illumination, position, flicker, and others, to visualize not only occupied but also unknown and free spaces within the urban environment, based on the obtained voxel occupancy probabilities.

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