Optimized Factor Graph for Tightly-Coupled LiDAR/IMU Localization in Underground Parking Garages 16787 STC IV Mid-term Symposium "Spatial Information to Empower the Metaverse", 22–25 October 2024, Fremantle, Perth, Australia

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Abstract

To address the challenge of intelligent vehicle localization in underground parking structures due to the loss ofGNSS signals, this paper introduces a method to address this issue by developing a novel localization framework known as GF-LIO, which denotes a tightly-coupled fusion of LiDAR and IMU data, innovatively combining the Interactive Extended State Kalman Filter (IESKF) with a factor graph to enhance the localization process and solve the problem of GNSS signal loss in underground parking lots. The GF-LIO model commences with a strategic feature selection process, facilitated by a greedy algorithm that prioritizes environmental cues within the point cloud data. This method effectively filters out redundant features, thereby enhancing the saliency of retained features and subsequently improving the robustness of the localization process. Following feature selection, the model integrates LiDAR and IMU measurements utilizing the IESKF algorithm, ensuring a cohesive fusion of sensor data and bolstering attitude estimation accuracy. The culmination of the GF-LIO framework involves factor graph optimization, a sophisticated technique that synthesizes LiDAR odometry, IMU pre-integration factors, and loop closure detection factors. This optimization step enhances the overall precision and consistency of the localization process, resulting in superior performance compared to existing methodologies. Experimental evaluations conducted within underground parking environments corroborate the efficacy of the GF-LIO model. Comparative analyses against established approaches such as A-LOAM, LeGO-LOAM, LIO-LOAM, and FAST-LIO demonstrate a notable performance improvement exceeding 12.53%. The proposed model adeptly integrates domain-specific environmental characteristics with multi-sensor data, thereby facilitating precise localization and map construction tasks for intelligent vehicle navigating within the intricate confines of subterranean parking structures.

1. Introduction

Precise localization technology is crucial for enabling autonomous driving assistance systems, particularly in navigating complex urban environments. Current outdoor garage automatic parking systems typically rely on Inertial Measurement Unit (IMU) assistance alongside Global Navigation Satellite System (GNSS) to ensure continuous and reliable positioning, mitigating the challenges posed by GNSS accuracy degradation due to multipath effects in urban settings (Lategahn and Stiller, 2014;Ahmed et al., 2018). However, underground parking lots present a unique challenge as GNSS signals are inaccessible, necessitating alternative indoor positioning methods such as Bluetooth (Faragher and Harle, 2015), Wi-Fi (Yang and Shao, 2015), and Ultra-Wideband (UWB) (Li et al., 2009). Nevertheless, these methods often entail high deployment and maintenance costs due to the requirement for base stations, limiting their applicability in intelligent vehicle positioning systems. Consequently, improving navigation and positioning accuracy in underground parking scenarios remains imperative.

Simultaneous Localization and Mapping (SLAM) technology, leveraging cameras and LiDAR as primary sensors, offers a promising solution by enabling concurrent localization and scene model reconstruction. This approach facilitates the creation of a three-dimensional spatial framework within enclosed underground parking lots, providing real-time and robust support for safe automatic parking operations (Li et al., 2024). For instance, A global semantic map has been developed by detecting road instances such as curves and speed bumps in LiDAR/IMU coupling to enhance attitude estimation robustness. parking lots, with this information being integrated with wheel encoder data for precise positioning(Yan et al., 2021) . However,

visual features are susceptible to variations in environmental lighting conditions, and their accuracy relies on the availability of sufficient landmarks. Many researchers have thus turned to LiDAR-based SLAM methods for underground parking lot positioning. An Inertia-Enhanced Generalized Iterative Closest Point (G-ICP) method(Li et al., 2018) was utilized, leveraging a multi-state Extended Kalman Filter to loosely couple LiDAR with IMU for the collaborative positioning of multiple vehicles in parking lots. Similarly,Parametric maps were constructed using horizontal and vertical geometric parameters and integrated into an online filter to estimate map parameters and localize vehicles within indoor parking lots(Han et al., 2018). Although LiDAR-SLAM positioning methods for automatic parking systems in underground garages are an active area of research, existing LiDAR-inertial fusion approaches require further refinement to adapt to the unique challenges of underground parking environments and enhance automatic parking accuracy. Challenges include managing large data volumes and achieving real-time performance in multi-source data fusion, addressing robustness issues during vehicle manoeuvring within parking lots, and improving feature matching precision. To tackle these challenges, this paper proposes a factor graph-optimized LiDAR/IMU tightly-coupled Integrated Enhanced State Kalman Filter (IESKF) fusion model named GF-LIO for underground parking lot entry positioning. Initially, the model identifies and discards degraded point cloud features from the underground parking environment while retaining high-quality features to reduce redundancy. Subsequently, the IESKF algorithm is employed for tight Finally, through the integration of IMU pre-integration factors and loop closure detection factors, factor graph optimization

updates LiDAR odometry pose, thereby improving overall positioning accuracy and consistency. This approach effectively integrates environmental characteristics of underground parking $\frac{1000 \text{ Hz}}{1000 \text{ Hz}}$ lots, consolidates multi-sensor data, and enhances pose Keyframe estimation precision and stability.

2. Methodology

The overall framework of the GF-LIO model is depicted in Figure 1. This model is designed to achieve precise localization in challenging environments such as underground parking garages. It consists of several essential components, including data preprocessing, Integrated Enhanced State Kalman Filter (IESKF) odometry, factor graph global pose optimization, and loop closure detection. The front-end laser-inertial odometry relies on the tight coupling of IESKF, while the back end integrates various factors using a factor graph. The key steps involved in the GF-LIO model are outlined as follows:

1. Data Collection and Preprocessing: Utilize collected IMU acceleration and gyroscope data to estimate the vehicle's motion state and correct LiDAR point cloud distortion caused by vehicle motion through IMU pre-integration.

2. High-Quality Feature Extraction: Extract line and plane features from the point cloud based on curvature and identify degenerate features according to the feature vector. Replace degenerate features with predicted values and update features to obtain high-quality features.

3. Submap Update: Extract keyframes and construct submaps, conduct feature matching between frames and maps, and compute relative pose transformations.

4. IMU-LiDAR Tight Coupling Localization Based on IESKF: Fuse IMU and LiDAR observation data using IESKF to update the system state and output laser-inertial odometry pose factors.

5. Factor Graph Optimization: Select a keyframe-based Euclidean distance method for loop closure detection and jointly optimize IMU pre-integration factors and laser-inertial odometry factors within the sliding window during the same time segment as the keyframe to achieve the best estimation of the vehicle's pose state.

Figure 1. Overall Framework of the GF-LIO Model.

2.1 Vehicle Motion Model

In addressing vehicle motion in urban underground parking garage scenarios, we consider the presence of curved trajectories with slow speeds. Therefore, we adopt the Constant Turn Rate and Velocity (CTRV) model from the quadratic motion model. The CTRV model assumes that the vehicle moves along a straight path with constant turn rate and constant velocity. The schematic diagram of this model is provided below:

Figure 3. The CTRV model.

The state variables *x* of the vehicle can be represented as follows:

$$
x = [p_x, p_y, v, \psi, \psi']^T
$$
 (1)

The components of the equation represent the vehicle's position, velocity, yaw angle, and yaw rate in a two-dimensional plane. Discretizing the state variables over continuous time yields the state function in discrete time:

Figure 3. The CTRV model.
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\nvelocity, yaw angle, and yaw rate in a two-dimensional plane.
\nDiscretizing the state variables over continuous time yields the
\nstate function in discrete time:
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x_{k+1} = x_k + \begin{bmatrix} \frac{v_k}{\psi_k} (\sin(\psi_k + \psi_k \Delta t) - \sin(\psi_k)) \\ \frac{v_k}{\psi_k} (\cos(\psi_k + \psi_k \Delta t) + \cos(\psi_k)) \\ 0 \\ \psi_k \Delta t \\ 0 \end{bmatrix}
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 (2)
\nwhere, $\Delta t = t_{i+1} - t_i$. When the vehicle is in a straight-line
\ndriving state, The state function of yaw rate $\psi_k = 0$ can be expressed as:

where, $\Delta t = t_{i+1} - t_i$. When the vehicle is in a straight-line driving state, The state function of yaw rate $\psi_k^* = 0$ can be expressed as:

18 Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences, V-term Symposium "Spatial Information to Empower the Metaverse", 22–25 October 2024
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Taking into account the presence of noise errors during vehicle motion, we introduce straight-line acceleration noise v_a and yaw angle angular acceleration noise v_{ψ} in the CTRV model all follow a Gaussian distribution with a mean of zero, $(0, \sigma_a^2), v_{\psi} \sim \mathcal{N}(0, \sigma_{\psi}^2)$. So the state function with noise can **ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Informs (SPRS TC IV Mid-term Symposium "Spatial Information to Empower the Metaverse",
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2.2 Laser Point Cloud Feature Processing Model

The feature processing model is based on the unique linear and planar environmental characteristics of underground parking lots. It involves extracting point cloud features, processing feature degradation, enhancing high-quality features, optimizing degraded features, and improving the utilization rate of point cloud features.

2.2.1 Point Cloud Preprocessing: Point cloud data preprocessing primarily addresses the offset in point cloud data caused by high-speed movement and speed bump disturbances of vehicles in underground parking lots. Assuming the vehicle is moving at a constant speed, high-frequency IMU data is used to perform linear interpolation on the point cloud, transferring
the rotation quantity of the IMU between adjacent frames, and cloud set at time t $(t = i, i + 1, ..., j)$ is $\{F_e^t, F_{me}^t, F_p^t, F_{mp}^t\}$, the rotation quantity of the IMU between adjacent frames, and removing motion distortion through transformation matrices. Define the inertial coordinate system *I*, the LiDAR coordinate
system and each F_{mp} contains four planar points. Through selecting an
system *I_n* and the laser-inertial odometry coordinate system and each F_{mp} cont system *L*, and the laser-inertial odometry coordinate system *O*。Assuming that the acceleration and, angular velocity at time *t* between the start time *i* and end time *j* of the current laser point cloud frame are constant and reducedness 2.2 **Laser Point Cloud reature Processing Mode**

The feature processing model is based on the unique linear and

planar environmental characteristics of underground parking

lost. It involves extracting point cloud featur to the start and end times are denoted as *Lⁱ* and L_j .Integrating the IMU measurements between these two frames yields the corresponding estimated quantity x_i^o . transformation matrices.
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\nrating the IMU measurements between these two
\nlls the corresponding estimated quantity $\mathbf{x}^o_{l_i}$.

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\n $\mathbf{x}^o_{l_i} = \left[\mathbf{p}^o_{l_i} \quad \mathbf{v}^o_{l_i} \quad \mathbf{q}^o_{l_i} \quad \mathbf{b}^T_{l_i} \right]^T$

\n(5)

\n $\left[\mathbf{p}^o_{l_j} = \mathbf{p}^o_{l_i} + \sum_{i=1}^{j-1} \left(\mathbf{v}^w_{l_i} \Delta t + \frac{1}{2} (\mathbf{R}^w_i (\hat{\mathbf{a}}_i - \mathbf{b}_{\mathbf{a}_i}) - \mathbf{g}^o) \Delta t^2 \right) \right]$

\nWhere $\mathbf{X}^L_{(i, p_j)}$ corresponding to
\n $\mathbf{v}^o_{l_j} = \mathbf{v}^o_{l_i} + \sum_{i=1}^{j-1} \left(\mathbf{R}^o_i (\hat{\mathbf{a}}_i - \mathbf{b}_{\mathbf{a}_i}) - \mathbf{g}^o \right) \Delta t$

\n(6)

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\nSimilarly, the as constructed, repre-
\npartuated, repre-
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\n $\mathbf{q}^o_{l_j} = \mathbf{q}^o_{l_j} \otimes \prod_{i=1}^{j-1} \left[\frac{1}{2} (\hat{\mathbf{a}}_i - \mathbf{b}_{\mathbf{a}_i}) \Delta t \right]$

Where $p_{i_t}^{\circ}$, $v_{i_t}^{\circ}$, $q_{i_t}^{\circ}$ represent the position, velocity, and $\left|\left|\left(X_{(j,j)}^n - X_{(j,j)}\right)\right| - f_{i_t}\right|$ orientation in the world coordinate system at time *t*, respectively ; g^0 a represents the gravitational acceleration in

 $\begin{pmatrix} 3 \end{pmatrix}$ Based on the variations of the extrinsic parameters T_I^L between $(\psi)\Delta t$ represent the rotation matrices from the world coordinate the laser-inertial odometry coordinate system; R_t^W and R_t^Q system to the laser-inertial odometry coordinate system at time *t*. Example 10 Sciences, Volume X-4-2024
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 L and \mathbf{R}_i^0 matrices from the world coordinate

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0 \Box the IMU and the LiDAR, the pose T_j^i at time *j* relative to time *i* can be calculated through interpolation. Once the coordinate systems are aligned, the point cloud distortion correction is completed.

$$
T_I^L = \left[\begin{array}{cc} p_I^{LT} & q_I^{LT} \end{array} \right]^T \tag{7}
$$

$$
\boldsymbol{T}_j^i = \boldsymbol{T}_i^{\omega^{-1}} \boldsymbol{T}_j^{\omega}
$$
 (8)

 $\frac{k}{2} \left(-\cos(\psi_k + \psi_k \Delta t) + \cos(\psi_k)\right)$ $\left(\frac{1}{2}v_{a,k} \sin(\psi_k)(\Delta t)^2\right)$ to dynamically optimize the feature information matrix and $\left[\frac{v_k}{2}(\sin(\psi_k + \psi_k \Delta t) - \sin(\psi_k))\right] \left[\frac{1}{2}v_{ak}\cos(\psi_k)(\Delta t)^2\right]$ **2.2.2 Selection of High-Quality Features:** environment Amals of the Photogrammetry, Remote Sensing and Spatial Information Sciences, Volume X-4-

em Symposium "Spatial Information to Empower the Metaverse", 22-25 October 2024, Fremantities
 $x_{k+1} = x_k + \begin{bmatrix} v_k \cos(\psi) \Delta t \\ v_k \sin(\psi$ $\begin{bmatrix} 1 \ 1 \end{bmatrix}$ features are assessed, and a random greedy algorithm is applied $x_{k+1} = x_k + \begin{bmatrix} v_k \cos(\psi) \Delta t \\ v_k \sin(\psi) \Delta t \\ 0 \\ 0 \\ 0 \\ \end{bmatrix}$ the laser-inertial odometry coordinate system to the asserinertial odometry coordinate system to the asserinertial odometry coordinate system to the asserinertial $\frac{1}{2}v_{\infty}(\Delta t)^2$ a combinatorial optimization problem under technical ψ_k^{λ} Δt $\begin{bmatrix} \frac{1}{2}v_{\psi}(\Delta t)^2 \\ v_{\psi}(\Delta t)^2 \end{bmatrix}$ a combination constraints, actively selecting appropriate feature \downarrow 0 \downarrow \downarrow \downarrow \downarrow \downarrow subsets to enhance the accuracy and efficiency of the SLAM *x*_k + x_k + x_k + $\frac{1}{2}$ x_k **b** x_k **i** α **c** α **c** α **c** α *v* α **c** α **c** α *x* α **c** *v x x* α *x* $\sqrt{v_{a,k}\Delta t}$ (4) adjust the feature count. These transforms feature selection into system. Assuming that point p_i and its five adjacent points in the same vertical direction form a point set *S*, compute the curvature of point p_i : the point cloud distortion correction is
 $T_I^L = \begin{bmatrix} p_I^{LT} & q_I^{LT} \end{bmatrix}^T$ (7)
 $T_J^i = T_i^{(0)} T_j^{(0)}$ (8)
 High-Quality Features: environment

and a random greedy algorithm is applied

nize the feature information matrix *j S j i ⁱ ^c ^S r r*

$$
c = \frac{1}{|S| ||\mathbf{r}_i||} \sum_{j \in S, j \neq i} ||(\mathbf{r}_j - \mathbf{r}_i)||
$$
(9)

Given a smoothness threshold c_{th} , if the smoothness is greater than the threshold, the point cloud is considered as an edge point, forming the edge point set *F^e* , otherwise, it is considered as a planar point, forming the planar point set F_p . To improve efficiency, a certain number of planar points and edge points are extracted to form a new point set. Assuming the feature point **2.2.2 Selection of High-Quality Features:** environment features are assessed, and a random greedy algorithm is applied to dynamically optimize the feature information matrix and digust the feature count. These transforms *teatures are sassesse, and a random greecay argonium is applied
to dynamically optimize the feature information matrix and
adjust the feature count. These transforms feature selection into
a combination constraints, acti* where $F_{me} \subset F_e$, $F_{mp} \subset F_p$, each F_{me} contains two edge points, curvature of point p_i :
 $c = \frac{1}{|S||\mathbf{r}_i||} \sum_{j \in S, j \neq i} ||(r_j - r_i)||$ (9)

Given a smoothness threshold c_{ih} , if the smoothness is greater

than the threshold, the point cloud is considered as an edge

point, forming th edge point P_j in F_e^j , the nearest point P_i in F_e^j to P_j based on KD-Trees determined, along with the point p_n nearest to p_i among the adjacent laser beams, ensuring that the three points are not collinear. The association equation of this edge point is represented by the distance from the point to the line: f planar points and edge points are

it set. Assuming the feature point
 $(i+1,...,j)$ is $\{F_e^i, F_{me}^i, F_p^i, F_{mp}^i\}$,

each F_{me} contains two edge points,

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est point p_i in F_e^j to edge point set F_e , otherwise, it is considered
forming the planar point set F_p . To improve
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a new point set. Assuming the feature point
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nt set. Assuming the feature point
 $L_i + 1, ..., j$ is $\{F_e^f, F_{me}^f, F_{pp}^f, F_{mp}^f\}$,

each F_{me} contains two edge points,

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rest point p_i in $F_e^$ *i* planar points and edge points are
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 i + 1,..., *j*) is $\{F_e^l, F_{me}^l, F_p^l, F_{mp}^l\}$,

each F_{me} contains two edge points,

planar points. Through selecting an

rest point p_i $c = \frac{1}{|S||r_i||} \sum_{j \in S, j \neq i} ||(r_j - r_j)||$ (9)

thness threshold c_n , if the smoothness is greater

hold, the point cloud is considered as an edge

the edge point set F_e , otherwise, it is considered

int, forming the plana $c = \frac{1}{|S||\mathbf{r}_i||} \sum_{j \in S, j \neq i} |(r_j - r_j)|$ (9)
thness threshold c_n , if the smoothness is greater
hold, the point cloud is considered as an edge
the edge point set F_e , otherwise, it is considered
int, forming the plan $c = \frac{1}{\|S\|\|r_i\|} \sum_{j \in S, j \neq i} \| (r_j - r_i) \|$ (9)

soothness threshold c_n , if the smoothness is greater

reshold, the point idual is considered as an edge

ng the edge point set F_e , otherwise, it is considered

point, f $|S| ||r_i||_{j \in S, j \neq i}||^{(1)}$ \rightarrow $\gamma ||$

mess threshold c_{ih} , if the smoothness is greater

old, the point cloud is considered as an edge

he edge point set F_e , otherwise, it is considered

th, forming the planar point fold c_{ik} , if the smoothness is greater
int cloud is considered as an edge
int set F_e , otherwise, it is considered
the planar point set F_p . To improve
r of planar points and edge points are
coint set. Assuming the f *i* at time t $(t = i, i + 1, ..., j)$ is $\{F_e, F_m, F_p, F_m, F_p, F_m\}$, $e \in F_e, F_{mp} \subset F_p$, each F_m contains two edge points, F_{mp} contains four planar points. Through selecting an *i P j* in F_e' , the nearest point *P*_{*i*} i *Le* contains two edge points,

soints. Through selecting an

the point p_i in F_e^j to P_j based on

the point p_n nearest

ms, ensuring that the three

tion equation of this edge

om the point to the line:
 $X_{(i,p_n)}$ *L* μ *L* μ *i* and μ *n* and μ *n n* and μ *n i (i, n_n)* μ *(10)*
 L L i i, p_n n eppresent the *i-th* frame.
 L z L i, p_n Mare $\left(\sum_{m} L_i \right) = \left(\sum_{m} L_i \right)$ *, each* L_m contains two edge points,

and each F_{mp} contains four planar points. Through selecting an

ddge point P_j in F_e^j , the nearest point P_i in F_e^j to P_j based on

$$
x_{i_i}^o
$$

(5)
$$
d_L = \frac{\left[\left(X_{(j,p_j)}^L - X_{(i,p_i)}^L \right) \times \left(X_{(j,p_j)}^L - X_{(i,p_n)}^L \right) \right]}{\left| X_{(i,p_i)}^L - X_{(i,p_n)}^L \right|} \qquad (10)
$$

 $\mathcal{X}_{(i,p_j)}^{\mathcal{I}}$ $\mathcal{X}_{t+1} = \frac{1}{2} \left(\mathbf{R}_{t}^W (\hat{\mathbf{a}}_t - \mathbf{b}_{\mathbf{a}_t}) - \mathbf{g}^0 \right) \Delta t^2$ Where $\mathcal{X}_{(i,p_j)}^{\mathcal{I}}$ represents the line feature $(\vec{\omega}_i - \vec{b}_{\omega_i})^{\Delta t}$ g^o) Δt corresponding to the *j-th* frame, and $X^u_{(i,p_i)}$, $X^u_{(i,p_n)}$ represent (6) the line feature coordinates corresponding to the *i-th* frame. *L* constructed, representing the distance from the point to the plane: not collinear. The association equation of this edge
presented by the distance from the point to the line:
 $L = \frac{\left| \left(\boldsymbol{X}_{(j,p_j)}^L - \boldsymbol{X}_{(i,p_i)}^L \right) \times \left(\boldsymbol{X}_{(j,p_j)}^L - \boldsymbol{X}_{(i,p_n)}^L \right) \right|}{\left| \boldsymbol{X}_{(i,p_j)}^L - \boldsymbol{X}_{(i,p_n)}^L \right|}$ $\left| \frac{X_{(i,p_i)}^L - X_{(i,p_i)}^L - X_{(i,p_n)}^L}{X_{(i,p_i)}^L - X_{(i,p_n)}^L} \right|$ (10)
 $\left| \frac{X_{(i,p_i)}^L - X_{(i,p_n)}^L}{X_{(i,p_i)}^L - X_{(i,p_n)}^L} \right|$

represents the line feature coordinates

ne *j*-th frame, and $X_{(i,p_i)}^L$, $X_{(i,p_i)}^L$ represent

oo *h* h collinear. The association equation of this edge
 $L = \frac{\left| \left(\mathbf{X}_{(j,p_j)}^{L} - \mathbf{X}_{(i,p_j)}^{L} \right) \times \left(\mathbf{X}_{(j,p_j)}^{L} - \mathbf{X}_{(i,p_n)}^{L} \right) \right|}{\left| \mathbf{X}_{(i,p_i)}^{L} - \mathbf{X}_{(i,p_n)}^{L} \right|}$ (10)
 $L = \frac{\left| \left(\mathbf{X}_{(j,p_j)}^{L} - \mathbf{X}_{(i,p_j)}$ not collinear. The association equation of this edge
resented by the distance from the point to the line:
 $= \frac{\left| \left(\mathbf{X}_{(j,p_j)}^L - \mathbf{X}_{(i,p_i)}^L \right) \times \left(\mathbf{X}_{(j,p_j)}^L - \mathbf{X}_{(i,p_n)}^L \right) \right|}{\left| \mathbf{X}_{(i,p_i)}^L - \mathbf{X}_{(i,p_n)}^L \right|}$ *h* $\left(\frac{h}{h}h_{j}(h_{j})\right) \times \left(X^{L}_{(i,p_{i})}\right) \times \left(X^{L}_{(i,p_{j})}-X^{L}_{(i,p_{n})}\right)$ (10)
 $\left|X^{L}_{(i,p_{i})}-X^{L}_{(i,p_{n})}\right|$ (10)
 $\left|\frac{X^{L}_{(i,p_{i})}-X^{L}_{(i,p_{n})}}{X^{L}_{(i,p_{i})}-X^{L}_{(i,p_{n})}}\right|$ (10)

represents the line feature coordinates
 he $\left(X_{(i,p_i)}^L\right) \times \left(X_{(i,p_i)}^L - X_{(i,p_n)}^L\right)$ (10)
 $\left|X_{(i,p_i)}^L - X_{(i,p_n)}^L\right|$

epresents the line feature coordinates
 e_j-th frame, and $X_{(i,p_i)}^L$, $X_{(i,p_n)}^L$ represent

ordinates corresponding to the *i*-th frame.

cont *X*^{*X} mp* contains four pianar points. In
four *P i* m *F*_{*i*}^{*i*} to *P*_{*i*} in *F*_{*i*}^{*i*} to *P*_{*j*} based on
s determined, along with the point *P*_{*n*} nearest
mong the adjacent laser beams, ensuring that</sup> mined, along with the point p_n nearest

adjacent laser beams, ensuring that the three

Illinear. The association equation of this edge

ed by the distance from the point to the line:
 $\left(\frac{t}{(j,p_j)} - X_{(i,p_i)}^L\right) \times \left(X_{(j,p$

$$
d_h = \frac{\left| \left(\left(\mathbf{X}_{(j,j)}^h - \mathbf{X}_{(i,i)}^h \right) \times \left(\mathbf{X}_{(i,i)}^h - \mathbf{X}_{(i,n)}^h \right) \times \left(\mathbf{X}_{(i,i)}^h - \mathbf{X}_{(i,m)}^h \right) \right|}{\left| \left(\mathbf{X}_{(i,i)}^h - \mathbf{X}_{(i,n)}^h \right) \times \left(\mathbf{X}_{(i,i)}^h - \mathbf{X}_{(i,m)}^h \right) \right|} (11)
$$

ISPRS Annals of the Photogrammetry, Remote Se

ISPRS TC IV Mid-term Symposium "Spatial Information to Emp

Where $X_{(j,j)}^h$ represents the surface feature coordinate

corresponding to the *j-th* frame, $X_{(i,j)}^h$, $X_{(i,j)}$ **ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial In

***h x i*_(*j*,*j*) represents the surface feature coordinates and FAST-L
 *h i*_(*j*,*j*) represents the surface feature coordinates and FAST-SPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Science

ISPRS TC IV Mid-term Symposium "Spatial Information to Empower the Metaverse", 22–25 October

Where $X_{(j,j)}^h$ represents the surface feat and $X_{(i,m)}^{(h)}$ represent the surface **ISPRS Annals of the Photogrammetry, Remote**
 TC IV Mid-term Symposium "Spatial Information to El
 $X_{(j,j)}^h$ represents the surface feature coordin

nding to the *j-th* frame, $X_{(i,i)}^h$, $X_{(i,i)}^h$
 h , represent the **is PRS Annals of the Photogrammetry, Remote Sensing and Spatial In

STC IV Mid-term Symposium "Spatial Information to Empower the Metaverse**
 *ii**X***^h**</sup> and FAST-L
 *ii**X***^h** (1) Observations and FAST-L
 ii corresponding to the *i-th* frame. To assess the quality of features, let $N = |F_K|$ denote the total number of features, M denote the maximum number of selected features S_K , represent the set of high-quality features, and $f(\cdot)$ denote the mechanism
moment's state variable \tilde{x}_{i+1} : measuring feature attributes. The feature selection formula
based on parameter constraints is expressed as: **ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information S

ISPRS TC IV Mid-term Symposium "Spatial Information to Empower the Metaverse", 22-25 Oc

Where** $X_{(i,j)}^h$ **represents the surface feature coord** *K K ^K ^K* nals of the Photogrammetry, Remote Sensing and Spatial Information
Symposium "Spatial Information to Empower the Metaverse", 22-2!

ents the surface feature coordinates and FAST-LIO1(Xt

following parts:

the *j*-th frame **ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information**
 ISPRS TC IV Mid-term Symposium "Spatial Information to Empower the Metaverse", 22–25 O

Where $X_{(i,j)}^{h}$ represents the surface feature coor Where $X_{(j,j)}^h$ represents the surface feature coordinates

corresponding to the *j*-th frame, $X_{(i,j)}^h$, $X_{(i,j)}^h$ (1) Observation State

and $X_{(l,m)}^h$ represent the surface feature coordinates

corresponding to the

$$
\underset{S_K - F_K}{\text{arg}\max} f\left[\Lambda(S_K)\right] \text{subject to } |S_K| \le M \tag{12}
$$
\n
$$
\tilde{P}_{k+1} = F_k \tilde{P}_k F_k^T + B_k O B B_k^T
$$

Where $\Lambda(S_K)$ represents the information matrix of the set of In equation matrix, $\lambda_{\min}(\Lambda)$ denotes the minimum eigenvalue,

To find high-quality features in real-time, a random greedy algorithm is employed to improve search efficiency. A subset of randomly sampled points from the map associated with the current frame is selected, and the residuals of all features in the subset are computed to choose the optimal feature. Subsequently, updates are made to the three information matrices in the algorithm. The loop terminates when the computation time for selecting high-quality features exceeds the maximum allows and log det (A) represents the logarithmic determinant used as a consecutive metric indicator for adaptively changing M_o measurement from the maximum allowing the maximum allows the maximum al high-quality features. The size of the random subset is defined as $\frac{N}{M} \log \left(\frac{1}{\varepsilon} \right)$, where ε is the degeneracy factor. The time error state vect g feature attributes. The feature selection for
parameter constraints is expressed as:
 $\arg \max_{S_{\kappa} \subset F_{\kappa}} \int [\Lambda(S_{\kappa})]$ subject to $|S_{\kappa}| \leq M$
 (S_{κ}) represents the information matrix of the s
lity features. $f(\Lambda): tr(\Lambda)$ Where $\Lambda(S_{\kappa})$ represents the information matrix of the set of
high-quality features. $f(\Lambda) : tr(\Lambda)$ denotes the trace of the
addition, $f(\bar{x})$,
matrix, $\lambda_{\min}(\Lambda)$ denotes the minimum eigenvalue, projects the sta
and log $\left(\frac{1}{\varepsilon}\right)$, independent of *M*. $A_{i+1}(t)$ $f[\Lambda(S_k)]$ subject to $|S_k| \le M$ (12)

ents the information matrix of the set of
 $f(\Lambda):tr(\Lambda)$ denotes the trace of the

denotes the minimum eigenvalue,

denotes the minimum eigenvalue,

features in real-time, a random greedy
 Assuming $f(\cdot)$ is a non-negative monotonic submodular function, the size of the random subset is set to $\frac{N}{M} \log \left(\frac{1}{\varepsilon} \right)$. update the prior covariance matrix:
 $\delta \mathbf{r}_{\cdot} = \mathbf{r}^{\alpha+1} \cap \mathbf{r}^{\alpha}$, $\mathbf{r}^{\alpha} \cdot \mathbf{r}^{\alpha}$. . apaaro S_K^* represents the optimal set, and S_K^* is the result of the random greedy algorithm. The expected approximation of S_K^* is: ampled points from the map associated with the
 α is selected, and the residuals of all features in the noise matrix, Q is the noise
 α : severeted, and the residuals of all features in the apportmants when the pre pled points from the map associated with the

soleced, and the residuals of all features in the noise matrix, *Q* is the noise

soleced, and the residuals of all features in the absolute difference bupdates are made to **Matters in Featuring, a fundame in the case of equation (15),** F_{i+1} **i persons the petroletic of points from the map associated with the noise matrix, Q** is the noise covariance mat selected, and the residuals of all

$$
E\left(f\left[\mathbf{\Lambda}\left(S_{K}^{*}\right)\right]\right) \geq \underbrace{(1-1/e-\varepsilon)}_{\text{expected ratio}} f\left[\mathbf{\Lambda}\left(S_{K}^{*}\right)\right] \tag{13}
$$

Finally, based on the logarithm of the determinants computed from all feature sets, it is determined whether they are highquality features, and the map state is updated using these highquality features.

2.3 Coupled Localization Model Based on IESKF

An a priori map is established based on the LiDAR to determine the vehicle's pose information in the underground parking garage, and during the parking process, the pose is updated by matching frames to the map. In this paper, the IESKF is employed to achieve tight coupling between LiDAR and IMU. Compared to traditional Kalman filtering, this model utilizes a real-time linearized system, where the first-order partial derivative of the error state is closer to the true state. Moreover, during the optimization process, the error rotation variable is approximated to zero, avoiding gimbal lock phenomena, reducing computational complexity, and minimizing errors. The overall framework is proposed based on LINS(Qin et al. ,2019)

^h ^Xi i , (,) **Example 3** Sensing and Spatial Information Sciences, Volume
 Empower the Metaverse", 22–25 October 2024, Frem

and FAST-LIO1(Xu et al., 2021), ma

following parts:

(1) Observation State Prediction: Assumerates

variab and FAST-LIO1(Xu et al., 2021), mainly divided into the following parts: (1) Observation State Prediction: Assuming the system state variables $\vec{x}_i = \left[p^T, q^T, v^T, b_s^T, b_a^T, g^T\right]^T$ represent the pose change nation Sciences, Volume X-4-2024
2–25 October 2024, Fremantle, Perth, Australia
(Xu et al., 2021), mainly divided into the
State Prediction: Assuming the system state
 $T, q^T, v^T, b_g^T, b_a^T, g^T$ represent the pose change
fr Information Sciences, Volume X-4-2024
se", 22–25 October 2024, Fremantle, Perth, Australia
-LIO1(Xu et al., 2021), mainly divided into the
parts:
ation State Prediction: Assuming the system state
 $\vec{E}_i = \left[\vec{p}^T, \vec{q}^$ *x* all Information Sciences, Volume X-4-2024
x prese", 22–25 October 2024, Fremantle, Perth, Australia
T-LIO1(Xu et al., 2021), mainly divided into the
parts:
vation State Prediction: Assuming the system state
 $\tilde{x}_$ from the *i-th+1* frame data to the *i-th* frame data, including displacement, rotation, velocity, gyroscope bias, accelerometer bias, and gravity. Inputting the motion-compensated LiDAR point cloud, IMU measurements, and measurement noise \boldsymbol{u} *i*, $\boldsymbol{\omega}$ *i*, , $\boldsymbol{\omega}_i$, , with $\boldsymbol{\omega}_i = 0$, predicts the state variable $\tilde{\boldsymbol{x}}_i$ to obtain the next and FAST-LIO1(Xu et al., 2021), mainly divided into the
following parts:
(1) Observation State Prediction: Assuming the system state
variables $\tilde{\mathbf{x}}_i = [\mathbf{p}^T, \mathbf{q}^T, \mathbf{v}^T, \mathbf{b}_g^T, \mathbf{b}_g^T, \mathbf{g}^T]^T$ represe ion Sciences, Volume X-4-2024

25 October 2024, Fremantle, Perth, Australia

iu et al., 2021), mainly divided into the

ate Prediction: Assuming the system state
 T^T , \mathbf{v}^T , \mathbf{h}_s^T , \mathbf{g}^T , T^T represen **ation Sciences, Volume X-4-2024**

-25 October 2024, Fremantle, Perth, Australia
 Xu et al., 2021), mainly divided into the

State Prediction: Assuming the system state
 \overline{q}^T , \overline{y}^T , \overline{b}^T_s , \overline{b}^T_s , *i* on Sciences, Volume X-4-2024
 i 25 October 2024, Fremantle, Perth, Australia
 i a et al., 2021), mainly divided into the

atte Prediction: Assuming the system state
 r_i^T , v_i^T , b_k^T , b_k^T , b_k^T , b_k^T , and FAST-LIO1(Xu et al., 2021), mainly divided into the
following parts:
(1) Observation State Prediction: Assuming the system state
variables $\tilde{x}_i = [p^T, q^T, v^T, b_s^T, b_a^T, g^T]^T$ represent the pose change
from the $i\text{-$

$$
\breve{\mathbf{x}}_{i+1} = \breve{\mathbf{x}}_i \bigoplus \bigg[\mathbf{f} \big(\breve{\mathbf{x}}_i, \mathbf{u}_i, \mathbf{\omega}_i \big) \cdot \Delta \bigg] \tag{14}
$$

$$
\breve{P}_{i+1} = F_i \breve{P}_i F_i^{\mathrm{T}} + B_i \mathcal{Q} \mathcal{B} \mathcal{B}_i^{\mathrm{T}} \tag{15}
$$

f the addition, $f(\tilde{x}_i, u_i, \boldsymbol{\omega}_i)$ represents the state

value,

projects the state change of the system

das a consecutive LiDAR frames at times *i*

measurement error state. Δt is the IN

equation (15), \tilde{P}_{i+1 In equation (14), \oplus denotes the generalized addition, $f(\vec{x}_i, \vec{u}_i, \omega_i)$ represents the state transition matrix that projects the state change of the system variables between two consecutive LiDAR frames at times i and $i+1$, based on the measurement error state. Δt is the IMU sampling period. In equation (15), \overline{P}_{i+1} represents the predicted covariance matrix at time $i+1$, \mathbf{F}_i is the predicted state matrix at time *i*, \mathbf{B}_i is the noise matrix, *Q* is the noise covariance matrix. After each iteration, the absolute difference between the state variables and the previous predicted values is checked to see if it is smaller than a threshold value σ : $\overline{\mathbf{x}}_i$ $F_i \vec{P}_i F_i^T + B_i \vec{Q} \vec{B} \vec{B}_i^T$ (15)
 \oplus denotes the generalized

oresents the state transition matrix that

of the system variables between two

es at times *i* and *i*+1, based on the
 Δt is the IMU sampling p \vec{x}_i : \vec{x}_i : $\vec{x}_i = \begin{bmatrix} \vec{x}_i \\ \vec{x}_i \\ \vec{x}_j \end{bmatrix}$: $\Delta \vec{t}_i$ (14)
 $= F_i \vec{P}_i F_i^T + B_i Q B B_i^T$ (15)
 \oplus denotes the generalized

epresents the state transition matrix that

e of the system variables between two
 secutive LiDAR frames at times *i* and *i*+1, based constrained in the *i* and *i*+1, based construent error state. Δt is the IMU sampling periation (15), \vec{P}_{i+1} represents the predicted covariance in the *i*, \vec *i*_{*i*+1} = *x_i* ω ₁ + *B*_{*i*} *A j* (15) $\overline{P}_{i+1} = F_i \overline{P}_i F_i^T + B_i Q B B_i^T$ (15) equation (14), \oplus denotes the generalized dddition, $f(\bar{x$ the predicted state matrix at time *i*, \mathbf{B}_i is the

s the noise covariance matrix. After each

te difference between the state variables and

ted values is checked to see if it is smaller
 w_{σ} :
 $\left\|\vec{x}_{i+1}^{a+1$ he predicted state matrix at time *i*, B_i is the
is the noise covariance matrix. After each
the difference between the state variables and
ted values is checked to see if it is smaller
lue σ :
 $\left\|\vec{x}_{i+1}^{a+1} \odot \vec{x}_{$ AR frames at times *i* and *i*+1, based on the
or state. Δt is the IMU sampling period. In
 $+1$ represents the predicted covariance matrix
the predicted state matrix at time *i*, B_i is the
is the predicted state matr

$$
\left\| \vec{x}_{i+1}^{\alpha+1} \ominus \vec{x}_{i+1}^{\alpha} \right\| < \sigma \tag{16}
$$

If it is less than the threshold, then the Jacobian matrix at the error state vector $\delta x_{i+1} = 0$ for the $i+1$ frame is computed, $A_{i+1}(\delta\theta_{i+1}) = I - \frac{1}{2}\delta\theta$ represents the rotational part of the

 $\frac{N}{N}$ log $\left(\frac{1}{N}\right)$. update the prior covariance matrix: matrix that describes the change in the error state between the updated state and the previous state. Equation (18) is used to

Q is the noise covariance matrix. After each
\nsolute difference between the state variables and
\nedicted values is checked to see if it is smaller
\n1 value
$$
\sigma
$$
:
\n
$$
\left\| \vec{x}_{i+1}^{a+1} \odot \vec{x}_{i+1}^{a} \right\| < \sigma
$$
\n(16)
\nn the threshold, then the Jacobian matrix at the
\ntor $\delta x_{i+1} = 0$ for the *i+1* frame is computed,
\n $I - \frac{1}{2} \delta \theta$ represents the rotational part of the
\nccribes the change in the error state between the
\nand the previous state. Equation (18) is used to
\nroovariance matrix:
\n
$$
\delta x_{i+1} = \vec{x}_{i+1}^{a+1} \odot \vec{x}_{i+1}^{a} + J_{i+1}^{a} \delta x_{i+1}^{a}
$$
\n
$$
J_{i+1}^{\alpha} = \begin{bmatrix} A_{i+1} (\delta \theta_{i+1})^{-T} & 0 \\ 0 & I \end{bmatrix}
$$
\n
$$
\vec{P}_{i+1} = \begin{bmatrix} J_{i+1}^{\alpha} \end{bmatrix}^{-1} \vec{P}_{i+1} \begin{bmatrix} J_{i+1}^{\alpha} \end{bmatrix}^{-T} \qquad (18)
$$
\nUpdate: Based on the pose changes of adjacent
\nin equation (14), residual equations $f\left(X_{i+1}^{i}\right)$ of
\nues are introduced. Covariance matrices H_{i+1} are
\nboth the edge point set and the plane point set:

$$
\breve{\boldsymbol{P}}_{i+1} = \left(\boldsymbol{J}_{i+1}^{\alpha}\right)^{-1} \breve{\boldsymbol{P}}_{i+1} \left(\boldsymbol{J}_{i+1}^{\alpha}\right)^{-T} \tag{18}
$$

(2) Error State Update: Based on the pose changes of adjacent frames x_{i+1} as in equation (14), residual equations $f(X_{i+1}^i)$ of observation values are introduced. Covariance matrices H_{i+1} are calculated for both the edge point set and the plane point set: ppdated state and the previous state. Equation (18) is used to

update the prior covariance matrix:
 $\delta x_{i+1} = \vec{x}_{i+1}^{\alpha+1} \odot \vec{x}_{i+1}^{\alpha} + J_{i+1}^{\alpha} \delta x_{i+1}^{\alpha}$
 $J_{i+1}^{\alpha} = \begin{bmatrix} A_{i+1} (\delta \theta_{i+1})^{-T} & 0 \\ 0 & I \end{bmatrix}$ (17

If it is less than the threshold, then the Jacobian matrix at the
error state vector
$$
\delta x_{i+1} = 0
$$
 for the *i+1* frame is computed,
 $A_{i+1}(\delta \theta_{i+1}) = I - \frac{1}{2} \delta \theta$ represents the rotational part of the
matrix that describes the change in the error state between the
updated state and the previous state. Equation (18) is used to
update the prior covariance matrix:

$$
\delta x_{i+1} = \bar{x}_{i+1}^{a+1} \delta x_{i+1}^a + J_{i+1}^a \delta x_{i+1}^a
$$

$$
J_{i+1}^{\alpha} = \begin{bmatrix} A_{i+1} (\delta \theta_{i+1})^{-T} & 0 \\ 0 & I \end{bmatrix}
$$
(17)

$$
\bar{P}_{i+1} = (J_{i+1}^{\alpha})^{-1} \bar{P}_{i+1} (J_{i+1}^{\alpha})^{-T}
$$
(18)
(2) Error State Update: Based on the pose changes of adjacent
frames x_{i+1} as in equation (14), residual equations $f(X_{i+1}^i)$ of
observation values are introduced. Covariance matrices H_{i+1} are
calculated for both the edge point set and the plane point set:

$$
\frac{\left\| \left(\bar{X}_{i+1,j}^{mc} - X_{i+1,j}^{mc} \right) \times \left(\bar{X}_{i+1,j}^{mc} - X_{i+1}^{mc} \right) \right\|}{\left| X_{i,j}^{kc} - X_{i+1}^{kc} \right|} \right\|}{\left| \left(\bar{X}_{i+1,j}^{mc} - \bar{X}_{i+1,j}^{mc} \right) \right| \times \left(\bar{X}_{i+1}^{mc} - \bar{X}_{i+1}^{mc} \right) \right|} \right\|_{\text{plane points}}}
$$
(19)

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\n**27RS TC IV Mid-term Symposium** "Spatial Information to Emperor the Metaverse", 22–25 October 2024, Female, Pe
\n*H_{r+1}* =
$$
\frac{\partial f}{\partial X_{(r+1,j)}^L} \frac{\partial X_{(r+1,j)}^L}{\partial \delta x} = \begin{cases} \frac{\left\{\left[\left(\frac{\tilde{X}_{(r+1,j)}^m - X_{(r+1,j)}^m}{\left[\left(\tilde{X}_{(r+1,j)}^m - X_{(r+1,j)}^m\right) \times \left(\tilde{X}_{(r+1,j)}^{m} - X_{(r,j)}^m\right)\right]^T\right]}{\left[\left[\left(\tilde{X}_{(r+1,j)}^m - X_{(r+1,j)}^m\right) \times \left(\tilde{X}_{(r+1,j)}^{m} - X_{(r,j)}^m\right)\right]^T\right]} \left[\mathbf{R}_{r+1}^i | \mathcal{X}_{(r+1,j)}^{m} \times \tilde{I} \right] & \text{(20)} \\ \frac{\left\{\left[\left(\tilde{X}_{(r,j)}^m - X_{(r,j)}^m\right) \times \left(\tilde{X}_{(r,j)}^{m} - X_{(r,j)}^m\right)\right]^T\right]}{\left[\left(\tilde{X}_{(r,j)}^m - X_{(r,j)}^m\right) \times \left(\tilde{X}_{(r,j)}^m - X_{(r,j)}^m\right)\right]^T\right]} \left[\mathbf{R}_{r+1}^i | \mathcal{X}_{(r+1,j)}^{m} \times \tilde{I} \right] & \text{(21)} \\ \frac{\left\{\left[\left(\tilde{X}_{(r,j)}^m - X_{(r,j)}^m\right) \times \left(\tilde{X}_{(r,j)}^m - \tilde{X}_{(r,j)}^m\right)\right]^T\right]}{\left[\left(\tilde{X}_{(r,j)}^m - X_{(r,j)}^m\right) \times \left(\tilde{X}_{(r,j)}^m - \tilde{X}_{(r,j)}^m\right)\right]^T\right]} \left[\mathbf{R}_{r+1}^i | \mathcal{X}_{(r+1,j)}^{m} \times \tilde{I} \right] & \text{(21)} \\ \frac{\left\{\left[\left(\tilde{X}_{(r,j)}^m - X_{(r,j)}^m\right) \times \left(\
$$**

He Photogrammetry, Remote Sensing and Spatial Information Sciences, Volume X-4-20

ium "Spatial Information to Empower the Metaverse", 22–25 October 2024, Fremantle,
 $\sum_{i=0}^{k} y_i \left(X_{i,j}^{m} - X_{i,j}^{m} \right)^T \left[\mathbf{R}_{i,j}^{i}(X_{$ Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences, Volume X-4-202
 i Symposium "Spatial Information to Empower the Metaverse", 22–25 October 2024, Fremantle, P
 $\left\{\frac{\left\{\left[\left(\frac{\chi_{m,i,j}^m - X_{(m,i$ **EXECTE ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information**
 EXECTE IV Mid-term Symposium "Spatial Information to Empower the Metaverse", 22–2
 Equation $H_{n,1} = \frac{\partial f}{\partial X_{0+1,0}^L} \frac{\partial X_{0+1,0}^$ coordinates of feature points in the edge point set F_{n_e} and the current pose estimate $T_{i+1} = \overline{T}_{i+1}$, The relative pose change plane point set F_{mp} , respectively, after motion compensation and high-quality feature selection between the *i-th* frame and the *ith+1* frame. Equation (20) where \mathbf{R}_{i+1}^{i} represents the pose change of the LiDAR point cloud between the *i*-th frame and the i -th+1 frame. $\begin{bmatrix} \end{bmatrix}$ denotes the skew-symmetric matrix transformation. $\frac{K_{i-1}}{2K_{i-1}} \frac{\partial Y_{i-1}}{\partial X_{i-1}} = \frac{K_{i-1} \sum_{i=1}^{n} \sum_{i=1}^{K_{i-1}} (X_{i-1}^{m} - X_{i-1}^{m}) \cdot (X_{i-1}^{m} - X_{i-1}^{m}) \cdot (Y_{i-1}^{m} - X_{i-1}^{m$ Example Solution Composition in the content poster and $\frac{24}{(1+\alpha_0+2\alpha_0+1)(\alpha_0+2\alpha_0+1)}$

($\frac{21}{(1+\alpha_0+2\alpha_0+1)(\alpha_0+2\alpha_0+1)}$) ($\frac{21}{(1+\alpha_0+2\alpha_0+1)(\alpha_0+2\alpha_0+1)}$) (21) optimal estimate is as follows:

(a) optimal $\frac{2x^2_{\text{max}}}{\sqrt[3]{x^2_{\text{max}}} + \frac{2x^2_{\text{max}}}{\sqrt[3]{x^2_{\text{max}} - x^2_{\text{max}}}} + \frac{2x^2_{\text{max}}}{\sqrt[3]{x^2_{\text{max}} - x^2_{\text{max}}}} + \frac{2x^2_{\text{max}}}{\sqrt[3]{x^2_{\text{max}} - x^2_{\text{max}}}} + \frac{2x^2_{\text{max}}}{\sqrt[3]{x^2_{\text{max}} - x^2_{\text{max}}}} + \frac{2x^2_{\text{max}}}{\sqrt[3]{x^2_{\text{max}} - x^$ $\begin{array}{llll} & \frac{\left\{\left[\left(X_{\ell_{i+1}^{m},j}-X_{\ell_{i+1}^{m}}^{m}\right)\times \left(X_{\ell_{i+1}^{m},j}-X_{\ell_{i+1}^{m}}^{m}\right)^{T}\right]\right\}}{\left[\left(X_{\ell_{i+1}^{m},j}-X_{\ell_{i+1}^{m}}^{m}\right)\times \left(X_{\ell_{i+1}^{m},j}-X_{\ell_{i+1}^{m}}^{m}\right)^{T}\right]}\left[\mathbf{R}_{\ell_{m}}^{U}(X_{\ell_{i+1}^{m},j})_{\ell}\right]^{T} & (20) & \text{optima} \\ & & \$ *i* μ_{i+1} in the dge point set $F_{\mu_{\text{w}}}$ and the current pose estimate T_{i+1} is in the dge point set $F_{\mu_{\text{w}}}$ and the current pose estimate T_{i+1} is in the deptection of the *i*-th frame and the *i*-

(1) re points in the edge point set F_m and the

expectively, after motion compensation and

selection between the *i*-*th* frame and the *i*-

on (20) where \mathbf{R}_{i+1}^i represents the pose
 $\begin{bmatrix}\n\mathbf{R}_{i+1}^i \text{ denotes the skew-symmetric matrix}$

Solving for the updated state quantity \bar{x}_{i+1}^{a+1} and the Kalman residual calculation gain K_{i+1} according to formulas (21) and (22):

$$
\widetilde{\mathbf{x}}_{i+1}^{\alpha+1} = \widetilde{\mathbf{x}}_{i+1}^{\alpha} - K_{i+1} f_{i+1}^{\alpha} - (I - K_{i+1} H_{i+1}) (J_{i+1}^{\alpha})^{-1} (\widetilde{\mathbf{x}}_{i+1}^{\alpha} \ominus \widetilde{\mathbf{x}}_{i+1}) (21)
$$
fa

$$
K_{i+1} = \left(\tilde{P}_{i+1}^{-1} + H_{i+1}^{\mathrm{T}} L_{i+1}^{-1} H_{i+1}\right)^{-1} H_{i+1}^{\mathrm{T}} L_{i+1}^{-1}
$$
 (22)

Correct the pose and output the posterior state quantity \hat{x}_{i+1} and the posterior covariance \hat{P}_{i+1} :

$$
\hat{x}_{i+1} = \breve{x}_i^{\alpha+1}
$$
 (23) repre

$$
\hat{P}_{i+1} = \left(I - K_{i+1} H_{i+1}\right) \tilde{P}_{i+1} \tag{24}
$$

formulas (21) and (22):

of three t

LiDAR-ir
 $\sum_{i+1}^{\alpha} - (I - K_{i+1}H_{i+1})(J_{i+1}^{\alpha})^{-1} (\tilde{x}_{i+1}^{\alpha} \odot \tilde{x}_{i+1})$ (21) factors an

for relativ

for relativ
 $\sum_{i+1}^{5-1} + H_{i+1}^{T}L_{i+1}^{-1}H_{i+1}^{-1}$ (22)

utput the post (3)Keyframe-Submap Matching: To avoid redundant information in the output of the IESKF for IMU-LiDAR odometry frame by frame, keyframes are extracted for processing, while discarding the remaining data. Considering that the vehicle's motion state includes stationary states, keyframe selection is set based on the significant position change of the set of high-quality feature collections exceeding 1m. Based on the current keyframe pose, *i* nearest keyframes **EXECT THEST** (EVALUATER THE TRANSFERIENT CONTINUOR EXECTS, LOOP clotted to $\tilde{x}_{i+1} = \tilde{x}_{i+1}^n - K_{i+1}f_{i+1}^n - (I - K_{i+1}H_{i+1})\left(J_{i+1}^{\alpha}\right)^{-1}\left(\tilde{x}_{i+1}^{\alpha}\right)^{-1}$ for relative pose between adjacent frames is:
 K_{i+1} corresponding poses are subsequently transformed to the coordinate system of the current keyframe F , forming a local submap, which is continuously updated with changes in the motion state. The Levenberg-Marquardt (LM) registration algorithm is used in this paper to match keyframes with local submaps. The cost function $f(\vec{F}_{i+1})$ is established using the point-to-line distance d_{\perp} and point-to-plane distance d_h (referred to collectively as d) between frames and submaps to solve for the optimal pose change relationship between frames and submaps:the permaining data. Considering estimation. The permaining data considering estimation. The permaining data considering estimation. The estimation on state includes stationary states, gyroscope in equality feature collec *F*, keyframes are extracted for

intily optimized with other characterial details at considering

tate includes stationary states, gyroscope in the IMU are as f

and the remaining details are as for a measurement and

di ectively as **d**) between frames and

optimal pose change relationship
 $\overline{F}_{i+1} = d$
 $\begin{bmatrix} d_L \\ d_h \end{bmatrix}$ (25)

Define the r

tionship between frames is:
 $\begin{bmatrix} 1 \\ 1 \end{bmatrix} + J(\overline{F}_{i+1})^T \Delta \overline{F}_{i+1}$ (26)
 $\begin{bmatrix} 260 \\$ d_{\perp} and point-to-plane the n

ectively as d) between frames and

optimal pose change relationship
 $(\overline{F}_{i+1}) = d$
 $= \begin{bmatrix} d_{\perp} \\ d_{\parallel} \end{bmatrix}$ (25)

Definitionship between frames is:

cons

proces
 $\left. +1 \right\rbrace + J(\overline{$ *Form in Set in the form of the CA*, and \vec{q} and \vec{q} are the measurement keyframe \vec{F} , forming a local
urrent keyframe \vec{F} , forming a local
urrent keyframe \vec{F} , forming a local
on \vec{F} , forming a lo **i** and its columerate. The Levenberg-Marquardt (LM) registration in the ration degriftm is used in this paper to match keyframes with local and *B* is the utubmaps. The cost function $f(\vec{F}_{i+1})$ is established using the

$$
\begin{cases}\nf(\overline{F}_{i+1}) = d \\
d = \begin{bmatrix} d_{L} \\ d_{h} \end{bmatrix}\n\end{cases}
$$
\n(25) Define the

The optimal pose change relationship between frames is:

$$
\begin{cases}\n\min \frac{1}{2} \left\| f(\overline{F}_{i+1}) + J(\overline{F}_{i+1})^{\mathrm{T}} \Delta \overline{F}_{i+1} \right\| & \text{integrate to compute the} \\
\text{s.t.} \left\| D \Delta \overline{F}_{i+1} < \mu \right\|_2 & \text{frame:} \\
\end{cases}
$$
\n(26) Integrate to compute the frames, obtaining the IN frame:

In the equation, J represents the Jacobian matrix, / $\partial \textbf{\textit{F}}_{i+1}$, where \boldsymbol{D} is the coef trust region radius.

 $\left\{ \left[(X_{(i+1)}^{m} - X_{(i+1)}^{m}) \times (X_{(i+1)}^{m} - X_{(i+1)}^{m}) \right] \right\} \right\}$ $\left[\mathbf{p}_{i \rightarrow (X^{mp}) \rightarrow I} \right]$ Given the damping factor λ , the iterative derivation of the (20) optimal estimate is as follows:

$$
\frac{\left\{\left[\mathcal{X}_{\mathbb{C},j}^{(x)}-\mathcal{X}_{\mathbb{C},j}^{(x)}\right)\times\left(\mathcal{X}_{\mathbb{C},j}^{(x)}-\mathcal{X}_{\mathbb{C},m}^{(x)}\right)\right]\right\}}{\left[\mathcal{X}_{\mathbb{C},j}^{(x)}-\mathcal{X}_{\mathbb{C},m}^{(x)}\right]}\left[\mathbf{R}_{i+1}^{(x)}\right]\left(\mathcal{X}_{\mathbb{C}^{i+1},j}^{(x)}\right)\times I}
$$
\n
$$
F_{i+1} = F_{i} - \left(\mathbf{J}_{i}^{\mathrm{T}}\mathbf{J}_{i} + \lambda \operatorname{diag}\left(\mathbf{J}_{i}^{\mathrm{T}}\mathbf{J}_{i}\right)\right)^{-1}\mathbf{J}_{i}^{\mathrm{T}}\mathbf{d}
$$
\n
$$
(27)
$$

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^{*i*}, 22–25 October 2024, Fremantle, Perth, Australia

amping factor λ , the iterative derivation of the

rate is as follows:
 $\vec{r}_{t+1} = \vec{F}_i - (\vec{J}_i^T \vec{J}_i + \lambda diag(\vec{J}_i^T \vec{J}_i))^{-1}$ Information Sciences, Volume X-4-2024
 Fe", 22–25 October 2024, Fremantle, Perth, Australia

damping factor λ , the iterative derivation of the

mate is as follows:
 $\mathbf{F}_{i+1} = \mathbf{F}_i - \left(\mathbf{J}_i^T \mathbf{J}_i + \lambda diag(\mathbf{J}_i^T$ Iterate continuously until convergence is achieved, obtaining the and Spatial Information Sciences, Volume X-4-2024
the Metaverse", 22–25 October 2024, Fremantle, Perth, Australia
Given the damping factor λ , the iterative derivation of the
optimal estimate is as follows:
 $F_{i+1} = F_i - ($ between two adjacent keyframes is:iciences, Volume X-4-2024

tober 2024, Fremantle, Perth, Australia

tor λ , the iterative derivation of the

llows:
 $I_i^T J_i + \lambda diag(I_i^T J_i)^{-1} J_i^T d$ (27)

l convergence is achieved, obtaining the
 $I_i = \overline{T}_{i+1}$, The relati

$$
\Delta F_{\text{max}} = F^{-1} F_{\text{max}} \tag{28}
$$

2.4 Factor Graph Optimization PoseModel

 $\left(I - K_{i+1}H_{i+1}\right)\left(J^a_{i+1}\right)$ $\left(\bar{x}^a_{i+1} \ominus \bar{x}_{i+1}\right)$ (21) factors and Odometry factors. The optimization objective term Utilizing the factor graph model computed by the front-end, residual calculation is performed and global optimization is conducted. The constraint relations in this paper are composed of three types of graph models: IMU pre-integration factors, LiDAR-inertial odometry factors, Loop closure detection for relative pose between adjacent frames is: club $(x_i^T J_i + \lambda diag(\boldsymbol{J}_i^T \boldsymbol{J}_i))^T \boldsymbol{J}_i^T \boldsymbol{d}$ (27)

til convergence is achieved, obtaining the
 $(\boldsymbol{J}_i^T \boldsymbol{J}_i + \lambda diag(\boldsymbol{J}_i^T \boldsymbol{J}_i))^T \boldsymbol{J}_i^T \boldsymbol{d}$ (27)

til convergence is achieved, obtaining the

reyframes is:
 ^L i i ^r x x x (29) where *^x* representing the state to be optimized; *^r* denoting the **i** $\binom{n}{i}$ $\binom{n}{i}$ $\binom{n}{i}$ chieved, obtaining the
ive pose change
(28)
Iodel
ted by the front-end,
 $\binom{n}{i}$ followed, optimization is
spaper are composed
re-integration factors,
is:
is:
is:
is:
is:
mized; r deno

$$
\phi_L(x) = \frac{1}{2} \| r(x_i, x_{i+1}) \|_{\Sigma}^2 \tag{29}
$$

 \hat{x}_{i+1} and where x representing the $\tilde{\mathbf{r}}^{a+1}$ (23) representing the motion states at time *i* and *i+1*; and $\tilde{\mathbf{r}}$ which residual of the relative pose factor; x_i, x_{i+1} respectively represents the covariance matrix of the constraint uncertainty.

(24) pre-integration of IMU data between adjacent keyframes **2.4.1 IMU Pre-integration Factors:** The high-frequency reduces the computational burden of multiple iterations while imposing constraints on IMU motion. These constraints are jointly optimized with other constraints to obtain the final pose estimation. The measurements from the accelerometer and gyroscope in the IMU are as follows: aph model computed by the front-end,
performed and global optimization is
int relations in this paper are composed
h models: IMU pre-integration factors,
try factors, Loop closure detection
factors. The optimization objec pose ractor; x_i , x_{i+1} respectively
states at time *i* and *i*+*I*; and *z* which
matrix of the constraint uncertainty.
ation Factors: The high-frequency
data between adjacent keyframes
is a burden of multiple iter *t* is performed and global optimization is
straint relations in this paper are composed
graph models: IMU pre-integration factors,
lometry factors. Loop closure detection
try factors. The optimization objective term
ween **Pre-integration Factors:** The high-frequency
 Pre-integration Factors: The high-frequency
 to f MU data between adjacent keyframes

mputational burden of multiple iterations while

straints on IMU motion. These const

$$
\widehat{\omega}_t = \omega_t + b_t^{\omega} + n_t^{\omega} \tag{30}
$$

$$
\hat{a}_t = \mathbf{R}_t^{\text{IW}} \left(a_t - g \right) + b_t^a + n_t^a \tag{31}
$$

 $\{\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_i\}$, the Where $\hat{\omega}_t$ and \hat{q} are the measurements of the IMU in its own coordinate system I at time t , influenced by the transformation biases b_i and white noise n_i , \mathbf{R}^{IW}_i is the rotation matrix from the IMU coordinate system *I* to the world coordinate system *W*, and *g* is the constant gravitational acceleration in the *W* coordinate system. *a* a^{*n*}_{*a*} (31)

of the IMU in its own

d by the transformation

he rotation matrix from

ld coordinate system *W*,

acceleration in the *W*

me Δt are derived from
 $\frac{a}{t} - n_t^a \Delta t$ (32)
 $\frac{a}{t} - b_t^a - n_t^a \Delta t^2$ c computational burden of multiple iterations while

constraints on IMU motion. These constraints are

limized with other constraints to obtain the final pose

. The measurements from the accelerometer and

in the IMU ar INC motion. These constraints are

net constraints to obtain the final pose

erments from the accelerometer and

as follows:
 $= \omega_t + b_t^{\omega} + n_t^{\omega}$ (30)
 $\sum_{i=1}^{m} (a_i - g) + b_i^{\omega} + n_t^{\omega}$ (31)

measurements of the IMU in it the transformation
the transformation
rdinate system W,
ration in the W
are derived from
 Δt
 Δt (32)
 Δt (33)
 Δt) (34) *t*₂ to the dial entwors and server and server and the computational burden of multiple iterations while g constraints on IMU motion. These constraints are popimized with other constraints to obtain the final pose on. T *theory* measurements from the accelerometer and
 t t $\hat{\omega}_t = \omega_t + b_t^{\omega} + n_t^{\omega}$ (30)
 $\hat{a}_t = \mathbf{R}_t^W (a_t - g) + b_t^{\omega} + n_t^{\omega}$ (31)

are the measurements of the IMU in its own

1 *I* at time *t*, influenced by the tran d with other constraints to obtain the final pose

reasurements from the accelerometer and

EMU are as follows:
 $\hat{\omega}_t = \omega_t + b_t^{\omega} + n_t^{\omega}$ (30)
 $\hat{a}_t = \mathbf{R}_t^W (a_t - g) + b_t^q + n_t^q$ (31)
 \hat{q} are the measurements of th gyroscope in the IMU are as follows:
 $\bar{\omega}_1 = \omega_r + b_r'' + n_r''$ (30)
 $\bar{a}_r = \mathbf{R}_r''' (a_r - g) + b_r'' + n_r''$ (31)

Where $\hat{\omega}_1$ and \hat{a}_1 are the measurements of the IMU in its own

coordinate system *I* at time *t*, influenced

The vehicle's velocity and attitude at time Δt are derived from the measurements:

$$
v_{t+\Delta} = v_t + g\Delta t + \mathbf{R}_t \left(\hat{a}_t - b_t^a - n_t^a \right) \Delta t \tag{32}
$$

$$
p_{t+\Delta t} = p_t + v_t \Delta t + \frac{1}{2} g \Delta t^2 + \frac{1}{2} \mathbf{R}_t \left(\hat{a}_t - b_t^a - n_t^a \right) \Delta t^2 \tag{33}
$$

$$
\mathbf{R}_{t+\Delta t} = \mathbf{R}_t \exp\left(\left(\widehat{\omega}_t - b_t^{\omega} - n_t^{\omega}\right)\Delta t\right)
$$
 (34)

(26) frames, obtaining the IMU state update equation for the $i+1-th$ constant angular velocity and acceleration during the integration process, according to the differential equation in equation (6), integrate to compute the IMU data between the *i-th* and *i+1-th* frame:

ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences, Volume X-4-4-4-S. T C IV Mid-term Symposium "Spatial Information to Empower the Metaverse", 22–25 October 2024, Fermant
$$
\left[P_{i+1}^W \right]
$$

\n
$$
\begin{bmatrix} p_{i+1}^W \\ p_{i+1}^W \\ q_{i+1}^W \end{bmatrix}
$$

\n
$$
\begin{bmatrix} p_{i+1}^W \\ p_{i+1}^W \end{bmatrix}
$$

\n
$$
\begin{bmatrix} p_{i+1}^W \\ p_{i+1}^W \end{bmatrix}
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\n
$$
\begin{bmatrix} p_{i+1}^W \\ p_{i+1}^W \end{bmatrix}
$$

\n
$$
\begin{bmatrix} p_{i+1}^W \\ q_{i+1}^W \end{bmatrix}
$$

\n
$$
\begin{bmatrix} q_{i+1}^W \\ q_{i+1}^W \end{bmatrix}
$$

\n
$$
\begin{bmatrix} q_{i+1}^W \\ q_{i+1}^W \end{bmatrix}
$$

\n
$$
\begin{bmatrix} 1 \\
$$

2.4.2 Laser-Inertial Odometry Factor: In order to obtain a more accurate pose transformation between two keyframes, the relative pose constraint relationship between LiDAR keyframes is utilized to optimize together with the IMU pre-integration factors in the sliding window. b_i^a
 b_j^a
 1 Factor: In order to obtain a
 1 between two keyframes, the

with the IMU pre-integration
 $\left(t_{i+1} - t_i\right)$
 $\left[\mathbf{R}_i^T \mathbf{R}_{i+1}\right]$
 $\left[\mathbf{R}_i^T \mathbf{R}_{i+1}\right]$ $\left[\frac{1}{2} \omega_i \Delta t \right]$ C; Rour
 b_i^a C; Rour
 ω_i^a is ω_i^b
 ial Odometry Factor: In order to obtain a

ransformation between two keyframes, the

nt relationship between LiDAR keyframes

ze together with the IMU p

$$
r_L = \begin{bmatrix} \Delta t - \mathbf{R}_i^{\mathrm{T}} \left(t_{i+1} - t_i \right) \\ \log \left(\Delta \mathbf{R}^{\mathrm{T}} \mathbf{R}_i^{\mathrm{T}} \mathbf{R}_{i+1} \right) \end{bmatrix} \tag{36}
$$

where **R** represents the rotation matrix in the Laser-Inertial Odometry.

2.4.3 Loop Detection Factor: In this paper, the method for loop detection adopts the keyframe-based Euclidean distance detection method proposed in LIO-SAM (Ye et al., 2019). Considering the short distance between adjacent keyframes of vehicle motion, to avoid redundant loop detections, odometer data is utilized to judge loops. Based on the translation information of keyframes, other keyframes within a 15-meter range and with time intervals greater than 30 seconds from adjacent keyframes are searched. After accumulating a certain number of waiting keyframes, historical keyframes that meet the conditions are identified for loop detection. Utilizing keyframes satisfying the conditions before and after the loop frame, a local map of the loop frame is constructed for frame-tomap matching. The relative pose change factor is solved through equation (28), and loop constraints are added for globalerror optimization to determine the optimal pose.

3. Experiments And Analysis

The experiments in this paper were conducted in one university underground parking. The wheeled mobile robot test platform was equipped with a Velodyne-16 LiDAR, operating at a sampling frequency of 10 Hz; the IMU used was the laboratorydeveloped Inertial-aided Pedestrian Navigation Module (IPMV), with a sampling frequency of 100Hz. The experimental site and the test platform are shown in Figure 4. There are various elements in the environment, such as speed bumps, drains, and uneven roads, which allow for testing the positioning accuracy and robustness of the algorithms.

(a) The Underground Parking (b) The wheeled mobile robot

Figure 4. Underground Garage Test Environment and Experimental Platform.

 $2^{2^{n-1}+q_1} \binom{m_1+m_2+q_2+q_3}{n_1}$ For the purpose of simulating underground parking garage 1 configured as follows: Route1: entrance to parking spaces A, B, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ route map and localization results are shown in Figures 5-6. $\beta_i^1 + \overline{a_i} \Delta t$ selected within the garage. The experimental routes were \mathcal{C}_i C; Route3: from the entrance, reversing to the exit. The driving (35) comiguited as follows: Route1: emailled to parking spaces A , B, C, and then to the exit; Route2: vehicles parking in spaces A , B, localization experiments, three parking spaces were randomly

Figure 5. Route Map.

Compared with traditional LiDAR SLAM algorithms such as A-LOAM (Ji et al., 2014), LeGO-LOAM (Shan and Englot, 2018), LIO-SAM (Meyers et al., 2020), and FAST-LIO2 (Xu et al., 2021) as illustrated in Figure 6.

(a) Localization results for Route1.

b) Localization results for Route2- Garage A.

(c) Localization results for Route2- Garage B.

(d) Localization results for Route2-Garage C.

(e) Localization results for Route3.

Figure 6. Localization results in the underground parking garage scene.

As shown in Figure 6, all five algorithms closely align with the ground truth trajectory. The red box marks the starting and ending positions of the vehicle, as well as its turning trajectory. However, A-LOAM and LEGO-LOAM, lacking IMU preintegration, exhibit larger errors. In autonomous parking scenarios, vehicles often require significant turning movements within a short period, emphasizing the importance of improving turning localization accuracy. During turning manoeuvres, GF-LIO utilizes loop closure detection for pose optimization, resulting in trajectories that closely follow the actual route. The loop closure detection in experiments A, B, and C in the parking garage is depicted in Figure 7.

Figure 7. Loop Closure Detection in Parking Garage Localization

The mean squared error comparison of the localization results for all routes between GF-LIO and traditional algorithms is presented in Table 1.

Table 1. Localization Results Comparison in Underground Parking Garage Scenarios. (RMSE: meters (m))

Drawing from the results of five experimental trials, the following observations can be made regarding the performance of the algorithm introduced in this study, GF-LIO, which is specifically designed for navigating environments within underground parking garages:

1. GF-LIO leverages an optimized fusion of LiDAR and IMU data through a factor graph approach, enabling the precise extraction of both planar features, such as walls and doors, and ground features, including parking bumpers and stationary vehicles. This leads to the generation of a detailed and comprehensive point cloud map of the underground parking garage environment.

2. The algorithm demonstrates a significant reduction in root mean square errors (RMSE) in both the horizontal and vertical planes when compared to traditional methods. More specifically, GF-LIO enhances positioning accuracy by 27.22%, 33.07%, 13.98%, and 12.53% respectively for Route 1; by 54.75%, 64.41%, 38.83%, and 26.48% respectively for Route 2 - Garage A; by 56.29%, 65.29%, 25.35%, and 20.95% respectively for Route 2 - Garage B; by 45.19%, 63.78%, 23.61%, and 18.03% respectively for Route 2 - Garage C; and by 40.15%, 61.71%, 36.32%, and 15.84% respectively for Route 2 overall. It is noteworthy that Route 2, which involves entry experiments in different garages over shorter distances, yields a higher number

of loop closures during the entry and exit phases, contributing to the notably enhanced accuracy. Routes 1 and 3 include sections where the vehicle enters and exits the garage, presenting substantial variations along the z-axis. Additionally, the segment from P22 to P23 encompasses speed bumps and ramps, as indicated by the relative error change plot for Route 1 in Figure 8, where an observable increase in error is noted for the P22 to exit segment.

4. Conclusion

This study addresses the challenge of GNSS signal unavailability in subterranean parking facilities by introducing a novel model known as GF-LIO. This model integrates the IESKF (Interactive Extended State Kalman Filter) with a factor graph for a tightly-coupled fusion of LiDAR and IMU data.GF-LIO model capitalizes on the rich environmental features for map matching to ascertain the relative pose of the LiDAR with precision, establishes an IESKF-driven LiDAR-inertial odometry system, and exploits IMU pre-integration and loop closure detection factors to enforce a unified constraint. Through the optimization of the factor graph, GF-LIO achieves a tight integration of LiDAR and IMU. Field entry tests in underground parking lots have shown that GF-LIO provides enhanced localization outcomes in structured settings. In the context of underground environments that present substantial challenges such as sharp turns and uneven surfaces, including speed bumps, GF-LIO surpasses conventional laser SLAM (IROS), Madrid, Spain, techniques in both accuracy and robustness.This model exhibits superior adaptability for the localization and mapping of mobile robots in intricate underground scenarios.

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