PERFORMANCE ANALYSIS OF GEOPOTENTIAL MODELS IN ESTIMATING THE GEOIDAL HEIGHT OVER IRAN

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KEY WORDS: Geopotential models, Geoidal height, geoid's potential, XGM2019e_2159, Iran

ABSTRACT:

Global geopotential models (GGMs) are widely used in earth sciences, especially in geodesy and geophysics. In this paper, we aim at studying GGMs' capability in the geoidal height estimation at the geographical region of Iran. This will include verification of both satellite-only and combined GGMs. To compute the geoidal height at a point, keeping the longitude and latitude constant, we change its ellipsoidal height until the GGM-derived gravity potential at the resulting height equals the geoid's potential value. In this process, we consider the topographic bias effect once the computation of the potential is inside the topographic masses. As the benchmark for the performance analysis of GGMs, we use the known geoidal heights of 841GPS/Levelling stations with a country-wide distribution. The results indicate that the XGM2019e_2159 (n_max=2190) GGM is the best combined and the GO_CONS_GCF_2_DIR_R5 (n_max=330) GGM is the best satellite-only model for the computation of geoidal heights in the study area, Iran.

1. INTRODUCTION

Nowadays, global geopotential models (GGMs) are widely used in engineering and geosciences. For instance, in topographic mapping via photogrammetry and remote sensing techniques, orthometric heights are required, and GGM can be the source to provide such information, in certain mapping scales. In some sciences such as geodesy, geophysics, and oceanography, GGMs are usually used as the reference field in the remove-restore technique for computation of the local geoid/quasigeoid, height datum unification, determination of internal structures of the earth, and computation of mean dynamic topography (MDT). Therefore, GGMs have been developed based on a variety of input data and methodologies. By increasing the quality and resolution of the observations obtained from satellite methods, terrestrial, airborne and ship borne methods, the resolution and accuracy of the GGMs have significantly improved. However, it should be noted that GGMs do not have a uniform accuracy in all regions due to heterogeneous terrestrial gravity data as well as different topography. GGMs are usually classified into two categories, i.e., satellite-only and combined models. The satellite-only models are developed by satellite data alone, while the combined models enjoy from both satellite and terrestrial data. In research projects, depending on the objective of the project, one may opt to use pure satellite models, or combined models.

In this paper, we aim at the performance analysis of different GGMs for the prediction of geoidal heights in the geographical region of Iran. For this purpose, we compute the geoidal heights at 841 GPS/Levelling stations using 27 different GGMs listed in Table 1 and compare the results with the geoidal heights derived from the GPS/Leveling. In this way, it will be possible for the users to choose suitable geopotential model for their application among the accuracy-sorted GGMs, both of the types combined and satellite-only, according to their performance in the estimation of geoidal heights in Iran.

2. METHODOLOGY

To compute the geoidal heights, we do not use common Bruns' formula. More specifically, the geoidal heights are derived from an iterative procedure starting with the computation of the gravity potential at the point of interest on the Earth's surface and then changing its ellipsoidal height in pre-defied decrements and finally computing the gravity potential at each step until we reach the geoid's potential value, W_0 (see Barthelmes, 2013). The reason for using this method is that there is no limit in choosing the reference ellipsoid. In this work, the geoid's potential value, W_0, is taken as 62,636,856.0 m^2/s^2 (Groten 2004). To explain the mentioned procedure more specifically let's refer to Figure 1. In Figure 1, the height of the geoid at the point P is shown by N and the point Q is the correspond point of P on the geoid. The potential of the point Q is equal to the geoid's potential value, W_0. The method used in this article is that in an iterative process by changing the height of the point from the ellipsoid surface, we reach a point whose potential is equal to W_0. In this case, the height of the point Q from the surface of the ellipsoid will be the same as the geoidal height.

The gravity potential, W, can be derived based on Eq. (1) (Heiskanen and Moritz, 1967):

$$W = V + V_c \tag{1}$$

where V is the gravitational potential and V_c is the centrifugal potential. The gravitational potential V can be computed from a GGM in terms of the spherical coordinates as follows (Heiskanen and Moritz, 1967):

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$$V(\lambda,\phi,r) = \frac{GM}{R}\sum_{n=0}^{n_{\max}}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}(\bar{C}_{nm}\cos(m\lambda)+\bar{S}_{nm}\sin(m\lambda))\bar{P}_{nm}(\sin(\phi)) \quad (2)$$

where R is the reference radius of GGM, GM is the geocentric gravitational constant, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the spherical harmonic coefficients, ϕ and λ are spherical latitude and longitude respectively, r is the radial distance, and $n_{\max}$ is the maximum degree of the GGM. The centrifugal potential V_c is computed from (Heiskanen and Moritz, 1967):

$$V_c(\phi,r) = \frac{1}{2}(r\cos\phi)^2\omega^2 \quad (3)$$

where ω is the angular velocity. It is important to note that Eq. (2) is the solution to Laplace's equation outside of the Earth's mass. Therefore, we cannot use Eq. (2) inside the topographic masses without considering the topographic bias effect (Sjöberg and Bagherbandi, 2011; Sjöberg, 2007; Sjöberg, 2009). The topographic bias effect on the gravitational potential is computed from the following formula (Sjöberg and Bagherbandi, 2011; Sjöberg, 2007; Sjöberg, 2009):

$$\delta V = \begin{cases} 2\pi G\sigma(H^2 + \frac{2H^3}{3R}) & \text{for } H > 0 \\ 0 & \text{for } H \le 0 \end{cases} \quad (4)$$

where H is the topographic height above the computational point, and σ is the Earth's surface mass density which is assumed to be constant equal to 2670 kg/m^3.

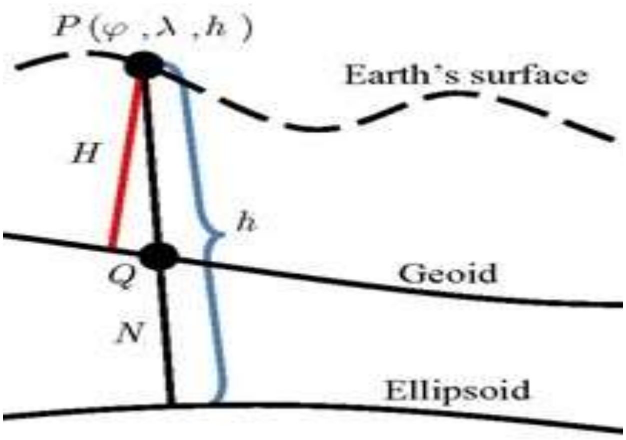

Figure 1. Definition of geoid height with respect to reference ellipsoid within the earth's body.

3. NUMERICAL RESULTS AND DISCUSSION

To evaluate different GGMs over Iran, we utilize 27 GGMs including both the combined and satellite-only GGMs. The GGMs applied in this study are listed in Table 1. We also use 841 GPS/Levelling stations as benchmark for the comparisons. Figure 2 shows the geographical distribution of 841 GPS/Levelling stations. Table 1, also summaries the results of the comparisons in terms of the maximum degree/order of GGM.

Model name	n_max	RMS (m)	RMS of EGM2008	RMS of EIGEN6c4	RMS of XGM2019e _2159
AIUB-CHAMP03S	100	1.49	1.493	1.497	1.497
ULux_ CHAMP2013s	120	1.293	1.217	1.226	1.226
EIGEN-2	140	2.332	1.081	1.079	1.078
EIGEN-GRACE02S	150	1.114	0.994	0.983	0.982
EIGEN-CHAMP05S	150	1.608	0.994	0.983	0.982
AIUB-GRACE03S	160	0.964	0.959	0.948	0.947
Tongji-Grace02s	180	0.819	0.815	0.802	0.801
GGM01C	200	0.819	0.735	0.721	0.722
GOCO01S	224	0.711	0.679	0.665	0.664
JYY_GOCE04S	230	0.655	0.669	0.655	0.653
GOCO03s	250	0.645	0.625	0.606	0.606
GO_CONS _GCF_2_ TIM_R5	280	0.578	0.559	0.54	0.54
ITU_ GGC16	280	0.578	0.559	0.54	0.54
GO_CONS_ GCF_2_ DIR_R5	300	0.566	0.518	0.498	0.498
GO_CONS _GCF_2_ SPW_R5	330	0.571	0.485	0.46	0.457
EIGEN-51C	359	0.48	0.453	0.429	0.424
EGM96	360	0.759	0.452	0.428	0.422
eigen-cg03c	360	0.622	0.452	0.428	0.422
EIGEN-GL04C	360	0.607	0.452	0.428	0.422
EIGEN-5C	360	0.609	0.452	0.428	0.422
GIF48	360	0.444	0.452	0.428	0.422
GGM05C	360	0.427	0.452	0.428	0.422
XGM2016	719	0.257	0.299	0.265	0.255
GOCO05c	720	0.302	0.300	0.266	0.255
EGM 2008	2190	0.268	0.268	0.219	0.207
EIGEN -6C4	2190	0.219	0.268	0.219	0.207
XGM2019e_2159	2190	0.207	0.268	0.219	0.207

Table 1. RMS of geoidal heights of 841 GPS/Levelling points calculated by all models.

https://doi.org/10.5194/isprs-annals-X-4-W1-2022-545-2023

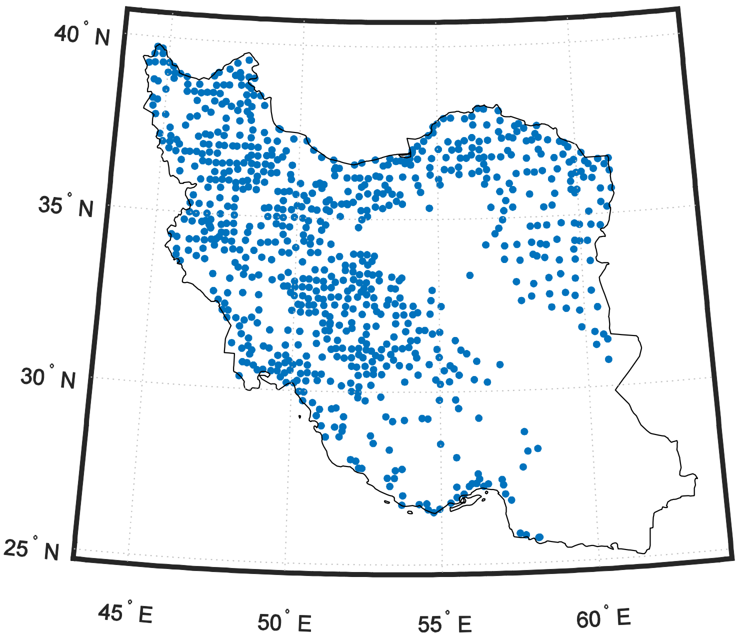

Figure 2. Geographical distribution of 841 GPS/Levelling points used in this study

From Table 1, it can be concluded that:
(1) Only the EGM2008 model enables to surpass the XGM2019e_2159 model at the degrees of 100 and 120, however their results are very close to each other. Beyond the degree of 120, the XGM2019e_2159 model is better than all models in almost all degrees. No combined model and no satellite-only model enable to surpass the XGM2019e_2159 model even in the low degree/orders. If we leave the XGM2019e_2159 model aside, except for the JYY_GOCE04S model at the degree/order of 230, the combined models are still better than the satellite-only models. This means that at the low degrees, the combined models calculate the geoidal height better than the satellite-only models.
(2) The comparison between the maximum degree of 2160 and 2190 can also be of interest to researchers. In the case of the XGM2019e_2159, EGM2008 and EIGEN6c4 models, the results show that there is no significant difference between the degree of 2160 and the degree of 2190. The differences between the degrees of 2160 and 2190 for the XGM2019e_2159 model at 841 GPS/Levelling points are plotted in Figure 3.

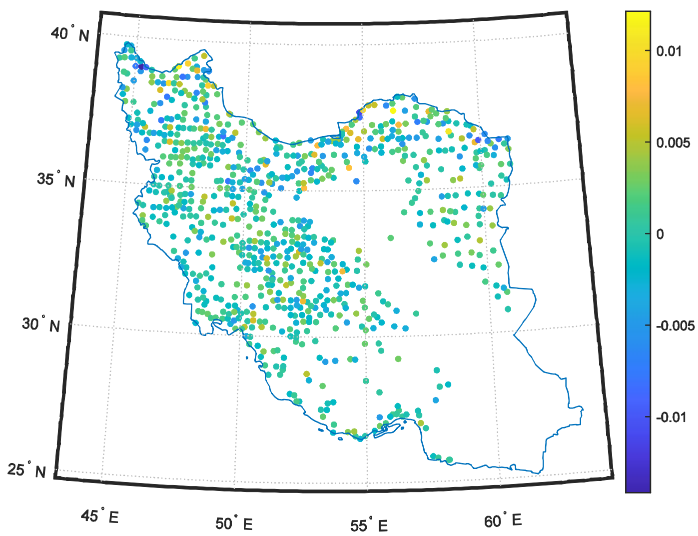

Figure 3. Differences between the geoidal heights computed up to the degree of 2160 and 2190 for the XGM2019e_2159 model at 841 GPS/Levelling points

In order to show the dependence of the accuracy of the GGM on the expansion degree, we depict the RMS of difference between the geoidal heights derived from the XGM2019e_2159 combined model and the GPS/leveling geoidal heights at different expansion degrees (Figure 4). From Figure 4, we can observe that the rate of accuracy improvement in low degrees is much higher than the rate of accuracy improvement in high degrees.

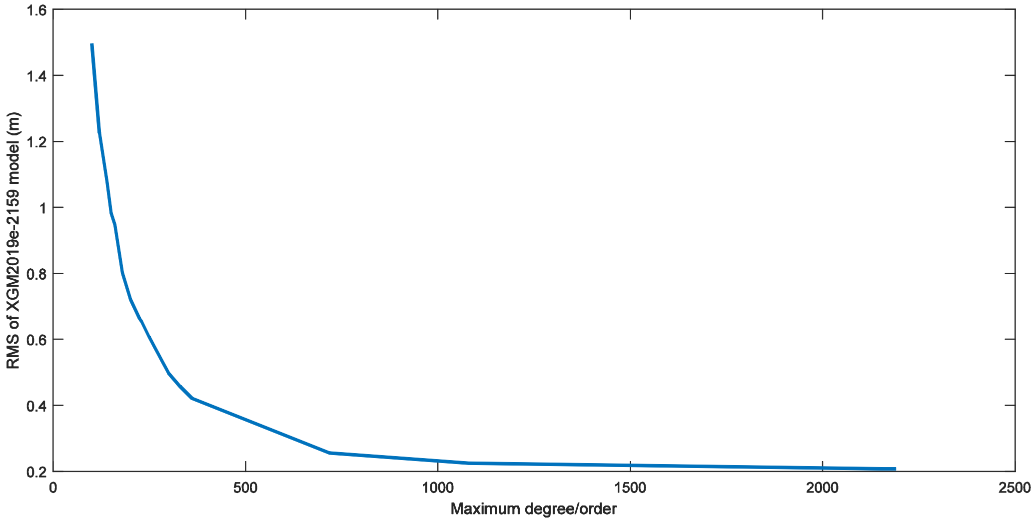

Figure 4. RMS of the XGM2019e_2159 model at different degrees

By sorting Table 1 based on the final accuracy of the GGM's, Table 2 is obtained.

Model name	type	n_max	RMS (m)
XGM2019e_2159	combined	2190	0.207
EIGEN_6C4	combined	2190	0.219
XGM2016	combined	719	0.257
EGM2008	combined	2190	0.268
GOCO05c	combined	720	0.302
GGM05C	combined	360	0.427
GIF48	combined	360	0.444
EIGEN-51C	combined	359	0.48
GO_CONS_GCF _2_DIR_R5	satellite-only	300	0.566
GO_CONS_GCF _2_SPW_R5	satellite-only	330	0.571
GO_CONS_GCF _2_TIM_R5	satellite-only	280	0.578
ITU_GGC16	satellite-only	280	0.578
EIGEN_GL04C	combined	360	0.607
EIGEN_5C	combined	360	0.609
Eigen_cg03c	combined	360	0.622
GOCO03s	satellite-only	250	0.645
JYY_GOCE04S	satellite-only	230	0.655
GOCO01S	satellite-only	224	0.711
EGM96	combined	360	0.759
Tongji_Grace02s	satellite-only	180	0.819
GGM01C	combined	200	0.819
AIUB_GRACE03S	satellite-only	160	0.964
EIGEN_GRACE02S	satellite-only	150	1.114
ULux_CHAMP2013s	satellite-only	120	1.293
AIUB_CHAMP03S	satellite-only	100	1.49
EIGEN_CHAMP05S	satellite-only	150	1.608
EIGEN_2	satellite-only	140	2.332

Table 2. Sorted GGMs according to the RMS of the differences between the geoidal heights computed via GGMs and the available geoidal heights of 841 GPS/Levelling stations in Iran.

According to the results shown in Table 2, the XGM2019e_2159 model (n_max=2190) is found as the most accurate combined model in the estimation of geoidal heights over Iran, and among the satellite-only models used in this study, the GO_CONS_GCF_2_DIR_R5 model has the first ranking, although its maximum degree/order is lower than some other satellite-only models.

As can be seen in Table 2, the accuracy of some satellite-only models such as the GO_CONS_GCF_2_DIR_R5 model, even though they have a lower degree, is better than the accuracy of some combined models such as the EIGEN_GL04C and EGM96 models.

Finally, we test the accuracy of XGM2019e_2159, EIGEN6c4, and EGM2008 models in the estimation of geoidal heights at different degrees, which are given in Table 3. The RMS values given in this table are the RMS of the difference between the computed and available geoidal heights at 841 GPS/Levelling stations.

Model name	RMS in n_max= 300 (m)	RMS in n_max= 360 (m)	RMS in n_max= 720 (m)	RMS in n_max= 1080 (m)	RMS in n_max= 2190 (m)
EGM 2008	0.518	0.452	0.300	0.281	0.268
EIGEN6 c4	0.498	0.428	0.266	0.236	0.219
XGM20 19e_ 2159	0.498	0.422	0.255	0.224	0.207

Table 3. RMS of geoid heights of 841 GPS/Leveling points calculated for most used combined geopotential models expanded to different degrees.

The RMS of the XGM2019e_2159 and GO_CONS_GCF_2_DIR_R5 models in 841 GPS/Levelling points of Iran up to 2190 and 300 degrees are plotted in Figures 5 and 6, respectively.

4. CONCLUSION

Among the combined models, the XGM2019e_2159 model is found as the most accurate combined GGM in the geoidal height estimation and among the satellite-only ones, the GO_CONS_GCF_2_DIR_R5 model attained the first rank.

Furthermore, our study showed that the XGM2019e_2159 GGM, when is expanded to degree 300 (Table 2) is still more accurate than all satellite-only models tested in this study, and also more accurate than the combined models EIGEN_GL04C, EIGEN_5C, Eigen_cg03c, and EGM96 up to their maximum degree of expansion, i.e., 360.

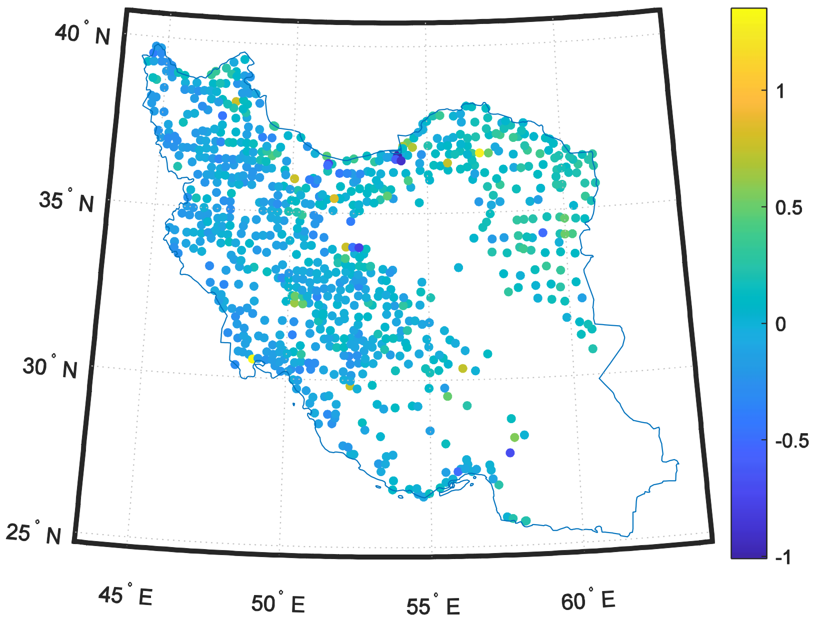

Figure 5. RMS of the XGM2019e_2159model in 841 GPS/Levelling points up to the degree of 2190.

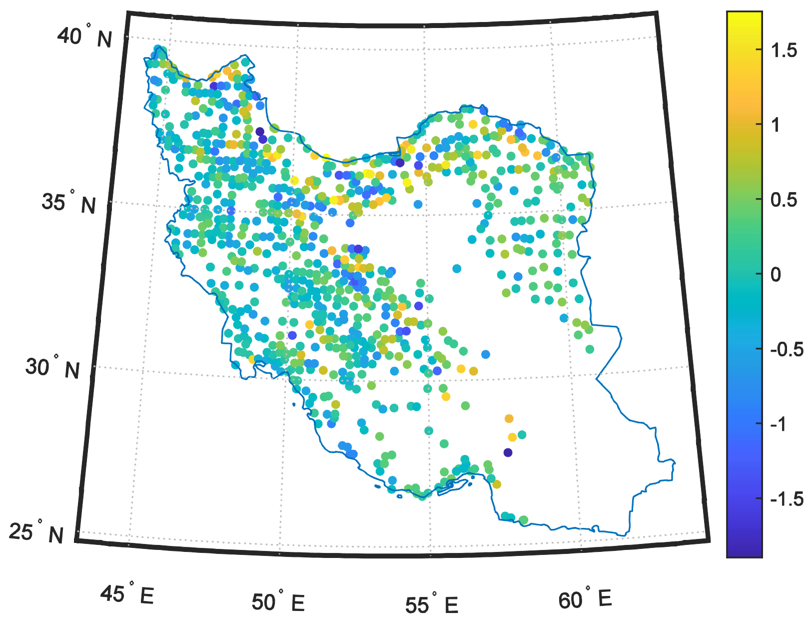

Figure 6. RMS of the GO_CONS_GCF_2_DIR_R5 model in 841 GPS/Levelling points up to the degree of 300.

ACKNOWLEDGEMENTS

National Cartographic Centre (NCC) of Iran is gratefully acknowledged for providing the GPS/Levelling data used in this study. The GGMs used in this study are obtained fromhttp://icgem.gfz-potsdam.de. We are grateful to the International Centre for Global Earth Models (ICGEM) for providing the GGMs and making them freely available.

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