SINGLE IMAGE SUPER RESOLUTION VIA COUPLED SPARSE AND LOW RANK DICTIONARY LEARNING

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Commission IV, WG IV/3

KEY WORDS: Image Super Resolution**,** Denoising, Sparse Representation, Dictionary Learning, Magnetic Resonance Imaging, Health Management, Remote Sensing

ABSTRACT:

Limitations in imaging systems and the effects of changes in sensing have caused limitation in acquiring high resolution images such as satellite images and magnetic resonance imaging (MRI). Sparsity can reduce the noises and improve the resolution. Super resolution in medical and satellite imagery is essential because low resolution image analysis is very difficult. Sparsity techniques have significant influence on computer vision specially when the main objective is extracting the meaningful information. The success of sparsity is related to the nature of signals such as image and sound which are naturally sparse because they were founded based on Wavelet and Fourier equations. In this research, we proposed a method for restoring a clear image from the related low-resolution parts of both MRI and satellite images. First, we proposed a widespread structure for learning the couple low rank and sparse main characteristic representation. Combined optimization of the nuclear and L_1 norms extracts the total low rank formation and the local patterns lodged in the image. In that case the reconstructed image will be more informative and matrix decomposition problem can recover a noisy observation matrix into an approximation of low rank matrix and a second matrix which contains some low dimensional structure. We assumed that by removing the blur and noise from these images, they would be reconstructed in the highest quality. The proposed method was compared with a variety dictionary learning approaches which addressed super resolution problem, such as tensor sparsity, Generative Bayesian and TV based methods. We demonstrated the results of applied method on MRI and satellite images, showing both visual and psnr improvements. Dealing with complex data in best manner shows the robustness of the proposed method.

1. INTRODUCTION

In the remote sensing, medical, military surveillance and reconnaissance and many other domains gaining high resolution (HR) images leads to huge consequence. However higher quality in imaging leads us to large and time-consuming problems. To address this problem, we proposed a super resolution approach using coupled dictionary learning incorporating dimension reduction strategy based on sparse and low rank structure. The key contribution of this work includes:

1.a new super resolution approach for low resolution (LR) satellite image and low resolution magnetic resonance imaging (MRI) are able to enhance various pairs of low- and high-resolution images

2. designing an effective coupled sparse dictionary, relying on alternative direction method due to the matrix-vector multipliers for preserving more spectral details

The rest of the article follows: section 2 provides an overview of the related articles. In section 3 the prepossessing and the SR based image on DL Scheme for the proposed method presents. Section 4 reports the experimental results, and conclusion of this work is presented in section5.

2. RELATED WORK

In super resolution (SR), problem sparsity plays an important role. Each signal is estimated by a linear combination of examples described as dictionary components, resulting in simple and compressed representation (Donoho,2006). Image patches can represent sparsely over a particular dictionary then some extra information are gained to enhance the visual quality of the recreated image. Sparsity techniques and dictionary learning (DL) have significant influence on computer vision specially when the main objective is extracting meaningful information.

https://doi.org/10.5194/isprs-annals-X-4-W1-2022-661-2023

Dictionary learning has been successful in solving inverse problems such as medical imaging, single pixel detector and background subtraction (Cevher et al.,2008; Egiazarian et al.,2007, Gong et al.,2015) predicting missing values in image inpainting and demosaicking. Dictionary learning break up the data matrix x into a dictionary matrix D and a presentation of matrix α, so that it's noted as matrix factorization.$x \approx D\alpha$

In other words, signal x in $\mathbb{R}^m$ is estimated with a sparse linear combination of a few columns of a matrix D in $\mathbb{R}^{m\times p}$. Various dictionary learning methods exist. Structured dictionary learning considers a particular structure among dictionary elements to overcome l_1-norm deficiency in modeling interactions among dictionary elements. Hierarchical dictionary learning considers a predefined hierarchical structure exist among dictionary elements. Zhou et al (2009) considered ψ as Group-Lasso penalty. A tree is described by the user and one dictionary component is dedicated to every node of the tree. Each group structure $\mathcal{G}$ of subsets of $\{1,\dots,P\}$including one node and all its descendants in the tree. The penalty function ψ is designated as $\psi(A) = \sum_{i=1}^{n} \sum_{g\in\mathcal{G}} \|\alpha_i[g]\|_q$.

In this tree structure dictionary components with low frequency gather near the root and high frequencies near the leaves. Inspired by topographic independent component analysis, Hyvarinen et al (2001) introduced a two-dimensional grid structure for dictionary learning. The main assumption is that dictionary elements can represent on a grid and neighbor relations between them are definable (Marial et al.,2011). Inspired by this method Szlam et al (2011) modeled inhibition impacts between dictionary components. Sahebi et al (2018) proposed a sliding window-based joint sparse representation model for hyperspectral anomaly detection.

Huang et al (2018) proposed an image super resolution algorithm based on an advanced sparse autoencoder. Ayas et al (2020) proposed a single image super resolution dictionary learning and sparse coding with multi directional Gabor feature representation. They used Gabor filter to extract image features at various scales and orientations. Therefore, the difficulty of capturing the complex local structures in all scales and variations is resolved. The effective mapping between low resolution and HR images achieves a high-quality reconstruction result. The learning-based dictionary has a comparatively powerful adaptive skill.

Therefore, Barman et al (2021) proposed a GPU accelerated adaptive dictionary learning and sparse representation for multispectral (MS) image super resolution. For edge preserving, they extracted high frequency characteristics presented in the input low resolution of MS image using Butter which worth low pass. Then various parallel algorithms are created for adaptive Dictionary learning.
In all of these applications choosing the proper dictionary is essential

3. PROPOSED METHOD

3.1. Data

Magnetic resonance imaging (MRI) of brain downloaded from the health section of https://www.data.gov/ in double complex format. IKONOS satellite image downloaded from ZENDOO dataset. This data contains 20% sampling of RGB, NIR and panchromatic bands.

3.2 Preprocessing of magnetic resonance imaging (pMRI)

The data acquiring level in conventional MRI is a relatively slow sampling process. To improve the scanning speed of MRI an efficient and feasible way is needed to acquire the data in parallel with multi-channel coils (Chen et al.,2013). The scanning time relies upon the number of measurements in the furrier domain and it will be considerably reduced when each coil only acquires a small fraction of the total measurements. In literature this issue called pMRI.

For furrier pulse sequences parallel imaging approaches invariably reduce the number of phase encoding steps needed to sample k-space and thereby decrease the imaging time. (Sahebkheir et al.,2019) In order to reduce the imaging time, MRI was reconstructed with pMRI based on the following formula: $for\ j = 1,\dots,J$
$RfS_ju = b_j + \eta$ where u is the unknown image, b_j is the vector of computed partial Fourier coefficients at the jth receiver, R is a diagonal sub-sampling operator, f is the Fourier transform, η is the Gaussian noise, and J is the total number of coils. The operator S_j is a diagonal matrix sensitivity mapping for the jth receiver, as it is used to refund for the crumble of signal intensity with distance from each pixel (Chen et al.,2013).

3.3. Image SR algorithm using dictionary learning

The super resolution problem can be defined as $Y = LS + n$. $S \in R^N$ is the HR image, $Y \in R^M$ is the LR image, L is the down sampling operator, n is as the additive noise. Expecting that there is an overcomplete HR dictionary $D_h \in R^{M\times N}$ and a LR dictionary $D_l \in R^{P\times N}$ sharing the same sparse coding. Then, the HR image S can be stated as a sparse linear combination of $\in R^n$, $S_h = D_h w$. The model of recovery solution for sparse coding is as follows:

$$min_w \|w\|_0 \quad subject\ to \quad \|S_l - D_l w\|_2^2 < \varepsilon \tag{1}$$

where ε notes as approximation error and $\|w\|_0$ denotes as l_0-psedu norm which computes the non-zero elements in a vector. l_0-psedu norm replaced with the l_1- norm because $l_1 = \sum_i |w_i|$ sparsify the solution and efficient optimization. Therefore, the optimization problem using Lagrange multiplier formulated as:

$$w^* = argmin_w \|S_l - D_l w\|_2^2 + p\|w\|_1 \tag{2}$$

where the parameter p controls the sparsity of the solution. By solving the equation (2) the sparse coefficient w can be obtained and the HR image is reconstructed by mapping onto the high-resolution dictionary $S_h = D_h w^*$.

3.4. Training set of preprocessing method

The training set containing 91 HR image derived from literature of (Huang et al.,2017). $S_h \in R^{P\times k}$ represents the HR images and the corresponding $S_l \in R^{M\times k}$ was constructed using scale down operator. Bicubic interpolation used to create related middle images S_m of the equal size as the HR images.

3.4.1. Create the HR training set

The HR images are deducted from middle images to eliminate their low frequencies. The difference images e_h can be obtained via $e_h = S_h - S_m$ (Zeyde et al.,2011). Then the HR training set can be gained by accomplishing feature extraction on different images e_h.

3.4.2. Create the LR training set

In order to pull out local features related to the high frequency contents r high pass filters conducted on S_m $\{R_i * S_m\}_i \quad , \ i = 1,2,\dots,r$ ($*$ denotes as convolution operator). Two kinds of high pass filters are preferred: gradient filters or Laplacian filters.

After this preprocessing step feature extraction is carried out on these filtered images. Then the LR training set z_l'can be achieved. Dimension of z_l' increased after operating an interpolation and linear filter. In that case, the last step before dictionary learning is dimensionality reduction. Sparse principal component analysis (SPCA) was conducted to reduce the dimension of z_l' and computation complexity. SPCA is based on PCA but it can provide more interpretable results. SPCA uses lasso constraint on the regression coefficient. This leads to representative and more accurate sparse principal components Merola (2014). Finally, the joint training set $z = [z_h, z_l]$ obtained by joining together the HR training set z_h with the LR training set z_l.

3.5. Coupled sparse dictionary learning

The image background generally has a low rank structure so the dictionary should have a low rank formation. Different objects can be distinguished with their specific characteristics. So, they should have sparse structure i.e., $rank(D_l) \leq k \quad , \|D_h\|_0 \leq s_1$. s and k are two integers representing the prior information on the upper bounds of the sparsity and the rank, respectively. The identity information of the same objects under different cameras is the same and should be as comparable as possible.

Therefore, the same objects should have the same coefficients even under different cameras because of that $w_h = w_l$. These methods rely on generating coupled sparse dictionary while jointly encoding two coupled feature spaces considering low rank information. Our aim is to discover a coupled dictionary pair D_l and D_h for the HR $S_h \in R^{M\times K}$ and LR $S_l \in R^{P\times K}$ images. The proper coupled dictionaries D_l and D_h can be approximated by solving the following sparse decompositions:

$$argmin_{D_h,D_l,W_h,W_l} \|S_h - D_h W_h\|_F^2 + \|S_l - D_l W_l\|_F^2 + \lambda_h \|W_h\|_1 + \lambda_l \|W_l\|_1 , \quad subject\ to\ W_h = W_l \quad \|D_h(:,i)\|_2 \leq 1 \quad , \|D_l\ (:,i)\|_2 \leq 1 \quad rank(D_l) \leq k \tag{3}$$

Where W_l is the sparse coefficient matrix related to LR image. W_h represents the sparse coefficients of HR image λ_h and λ_l controls the sparsity penalty. Alternative direction method of multipliers (ADMM) (Jiao et al.,2016) can convert the constrained dictionary learning problem represented in (3) into an unconstraint form. This can be formulated as bellow:

$$\begin{gathered}(D_h, W_h) = argmin\|D_h W_h - S_h\|_F + \lambda_h \|W_h\|_1 \\ (D_L, W_L) = argmin\|D_L W_L - S_L\|_F + \lambda_L \|W_L\|_1 , \\ \|D_h(:,j)\|_2^2 \leq 1 \ , \|D_l(:,j)\|_2^2 \leq 1 \quad and \\ W_h = W_l \quad , rank(D_l) \leq k\end{gathered} \tag{4}$$

For easier calculation the equations in (4) formulated as l_1-minimization problem:

$$\begin{gathered}min_{D_h,D_l,W_h,W_l} \|S_h - D_h W_h\|_F^2 + \|S_l - D_l W_l\|_F^2 + \lambda_h \|P\|_1 \\ subject\ to\ \ P - W_h = 0 , \quad Q - W_l = 0 \\ W_h - W_l = 0 \ \ \|D_h(:,i)\|_2 \leq 1 \quad , \\ \|D_l\ (:,i)\|_2 \leq 1 \quad \|D_h\|_{2,0} \leq s_1\end{gathered} \tag{5}$$

ADMM is a dominant algorithm for solving structural convex optimization problems (Boyd et al.,2011). ADMM uses augmented Lagrange so that the objective function of the original problem splits into several sub problems. The DL step for sparse representation of signals considering low rank information formulates as: $min_{D,X}\{\|Y - DX\|_F^2\}$, $s.t. \ \|x_i\| \leq k \ , \ i = 1,2,\dots,l$ where Y is the training matrix, D is the dictionary and X denotes the projection of the signals onto the dictionary D, k is the upper bound of the sparsity coefficients.

For solving this optimization equation ADMM and a given initial dictionary D and training matrix Y are used. Then OMP algorithm performed for the sparse coding stage. Therefore, coefficients X can be solved. By considering X is fixed, we updated dictionary D. Iteration continues till the convergence satisfies error of the signal representation. Indeed, DL based on ADMM algorithm changes the problem to this format using the Lagrange function: $l = \|Y - Z\|_F^2 + \sum_{i=1}^{l} < \Lambda_i , (Z - DX)_I > + \frac{\beta}{2}\|Z - DX\|_F^2$ where Λ_i is the ith column of Lagrange multiplier matrix.

The ADMM algorithm is applied to solve this equation and OMP algorithm is used to solve the coefficients of the equation and the updated dictionary obtained: $D^{(n+1)} = D^{(n)}(:,i) + \frac{H^{(n)}X^{(n)}(:,i)^T}{w^{(n)}+\delta}$
By setting ∇_{D_h}=∇_{D_l}=0 the high- and low-resolution dictionaries are updated column by column ensuing this repetitive procedure:

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$$\phi_h = W_h(:,j).W_h(:,j)^T \quad (6)$$
$$\phi_l = W_l(:,j).W_l(:,j)^T$$
$$D_h^{(k+1)}(:,j) = D_h(:,j)^{(k)}(:,j) + \frac{S_h.W_h(:,j)}{\phi_h + \delta}$$
$$D_l^{(k+1)}(:,j) = D_l(:,j)^{(k)}(:,j) + \frac{S_l.W_l(:,j)}{\phi_l + \delta}$$

k indicate the number of iterations, δ is a small regularization factor, $D_h(:,j)$ and $D_l(:,j)$ represents the jth column of D_h and D_l. Eventually, the Lagrange multiplier matrixes updates as:

$$Y_1^{(k+1)} = Y_1^{(k)} + c_1(P - W_h) \quad (7)$$
$$Y_2^{(k+1)} = Y_2^{(k)} + c_2(Q - W_l)$$
$$Y_3^{(k+1)} = Y_3^{(k)} + c_3(W_h - W_l)$$

The same as (Fotiadou et al., 2019) we set $c_1 = c_3 = 0.8$ and $c_2 = 0.6$

4. EXPERIMENTAL RESULT

In this section, we represent the results of the proposed method and its comparison obtained from ASDS Dong et al., (2011), Generative Bayesian SR of Zhang et al., (2012), Low Rank Tensor Completion by Liu et al., (2013), FISTA by Beck et al., (2009) and Sparse representation based iterative incremental image deblurring from Zhang et al., (2009) implemented on both satellite and MRI images.

First, we used pMRI technique for reconstructing magnetic resonance imagery in complex double type which was downloaded from https://www.data.gov/. (Sahebkheir et al.,2019) The reconstruction procedure took about 3 minutes implemented in Matlab 2016b on a desktop Intel corei7 8550U CPU. The result of pMRI is shown in Figure 1(a). After the reconstruction, some blurs appeared; this time we assumed that by performing super resolution algorithms and removing the blurs, the image should be reconstructed in the best of its quality. The result of pMRI is the test image for the deblurring and image super resolution step.

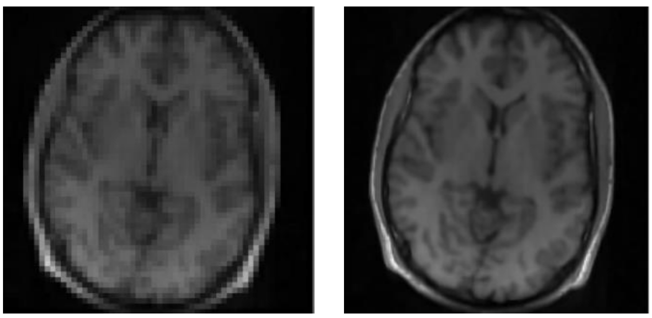
We studied several deblurring and SR methods based on sparse representation modelling and dictionary learning to find the best dictionary. Then the proposed method for image SR implemented in the same environment for both pMRI result and satellite image. The proposed methos uses coupled DL incorporating low rank information. The ADMM algorithm used for DL which was faster than the state-of-the-art DL methods used in validation algorithms such as K-SVD or BCD. The average running time of proposed method and its validations for MRI imaging and satellite image listed in table 1.

As it’s shown, the proposed method is the fastest. The sparse coding stage in the proposed method performed by OMP. For fair comparison, the proposed method and the validation algorithms used the same training set extended by 90° rotation considering the visual importance of edges. Degradation factor set $s = 3$ and white Gaussian noise with standard deviation of 2 were adopted for all algorithms. Feature extraction employed by Laplacian filters and gradient. Nearly 130,000 training patch pairs were gathered. In the proposed method, SPCA applied for dimensional reduction. For the proposed method, the HR dictionaryD_h and the LR dictionary D_l utilizing coupled sparse dictionary learning. Sparse representation coefficients calculated by OMP algorithm and the reconstructed HR image calculated via $S = D_h w$. The psnr (peak of snr) calculated by Eq (8):

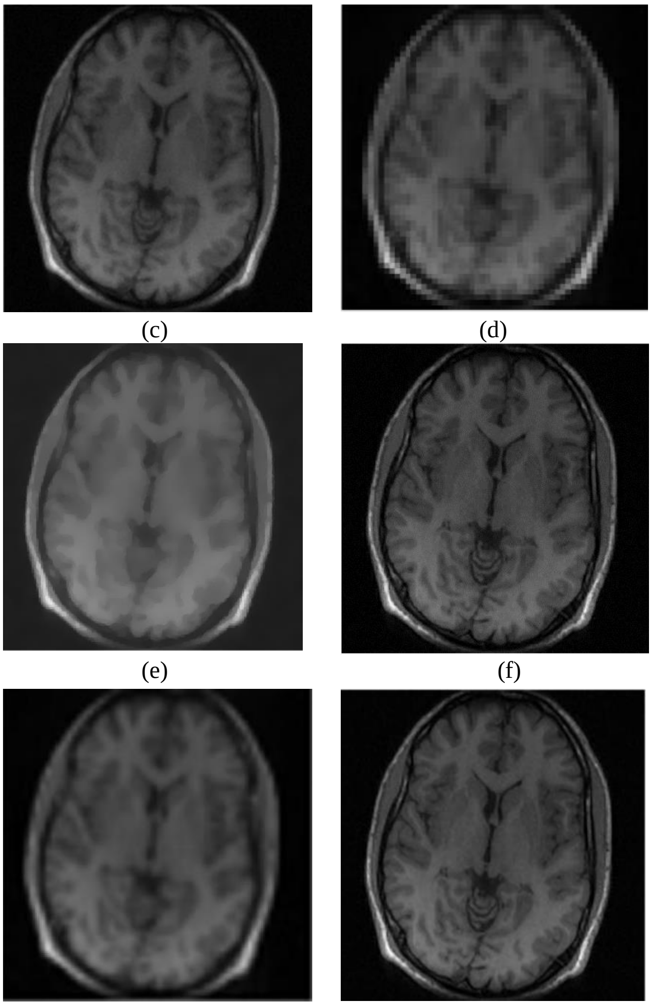
$$psnr = 10log_{10}\left(\frac{255^2 N}{\sum_i(\acute{S}_i - S_i)^2}\right) \quad (8)$$

Figure 1 illustrates the result of pMRI reconstruction and employment of proposed and validation methods on it. Figure b illustrates the result of SR algorithms on satellite image. Figure1(b) shows that ASDS suppresses the noise but some details are lost. Figure1(c) shows fine reconstruction result but it took 338.37minutes. In BCD method incorporating tensor sparsity leads to cause ghost artifacts enclosing the edges in figure1 (d) and figure 2 (e). TV based methods like FISTA are efficient in overpowering the noise, however, they produce oversmoothed results and exterminate much details represented in figure1 (e) and figure2. (f). The l_0-norm sparsity-based methods are effective in rebuilding smooth parts but it fails to rebuild sharp edges in figure1. (f). The proposed approach has the best visual quality and the distances between white matter and gray matter which are constructed accurately in figure1(h). Figure2. (c) shows that ASDS suppress the noise but produces piecewise constant block artifact. Figure2. (d) (f) (g) color distortion is obvious.

In Figure2 (e), the reconstructed image is jaggy and contains ringing effects. Figure2. (l) shows that the proposed approach has the finest observable quality and edges are preserved. Table2 represent the PSNR and SSIM related to MRI and satellite image reconstruction under different methods. The experimental results on both MRI and satellite image clarified that the proposed approach outperforms various state-of-the-art methods in both PSNR, SSIM and visual quality.



(a) (b)



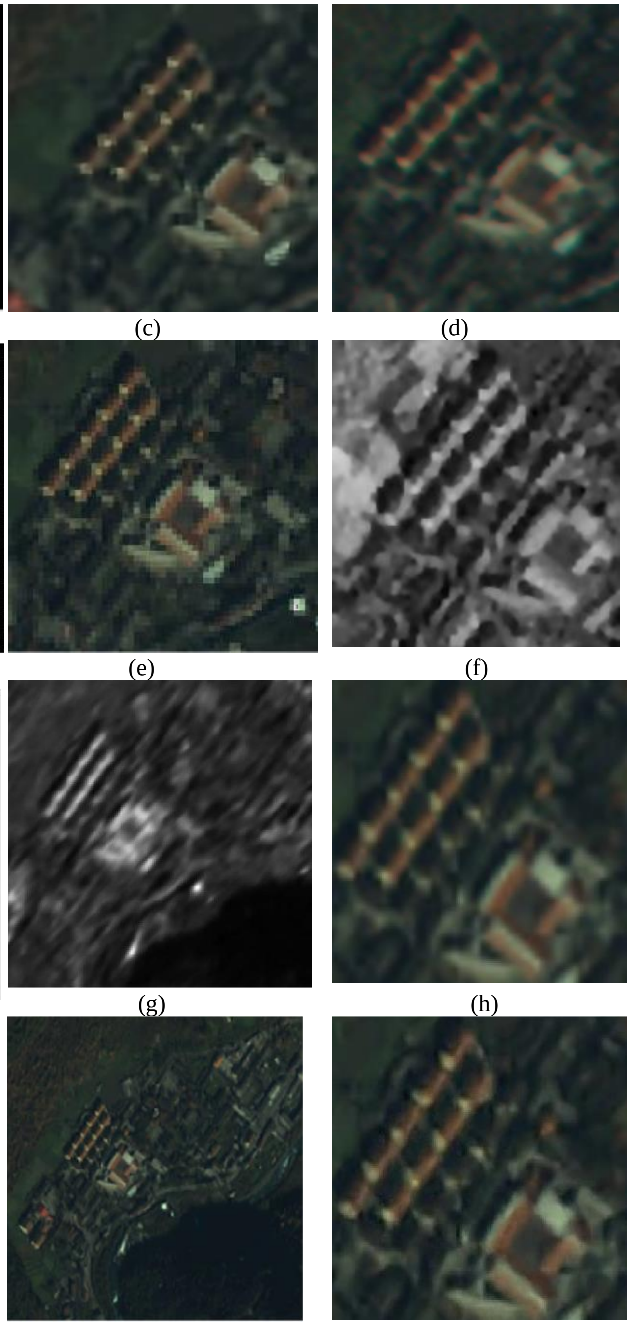
(c) (d)

(e) (f)

(g) (h)

Figure1. (a) The result of pMRI reconstruction, (b)ASDS, (c) Generative Bayesian, (d) Tensor Sparsity, (e) FISTA, (f) Incremental DL, (g) Bicubic, (h) Proposed Method

(a) (b)



(c) (d)

(e) (f)

(g) (h)

(k) (l)

Figure2. (a) original image, (b) 3times zoomed in, (c)ASDS, (d) Generative Bayesian, (e) Tensor Sparsity, (f) FISTA, (g) Incremental DL, (h) Bicubic, (k) Proposed Method, (l) 3times zoomed in proposed method's result

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Method	Running time of MRI imaging SR (minutes)	Running time of satellite image SR (minutes)
pMRI	3	----------
ASDS	12	17.2
Generative Bayesian	338.37	1065.1
Tensor Sparsity	45	51
FISTA	22	29
Incremental DL	48	53
Bicubic	0.75	1.1
Proposed Method	0.5	0.83

Table1. Represents running time of SR algorithms

Method	Psnr (satellite image)	Psnr (MRI imaging)	SSIM (satellite image)	SSIM (MRI imaging)
ASDS	36.07	32.3022	0.8117	0.8852
Generative Bayesian	33.36	32.12	0.8546	0.9051
Tensor Sparsity	19.62	24.36	0.659	0.8771
FISTA	22.24	27.26	0.671	0.8166
Incremental DL	22.41	29.04	0.678	0.9173
Bicubic	29.15	30.02	0.8554	0.8634
Proposed Method	42.63	34.15	0.9124	0.9393

Table 2. Represents the Psnr and SSIM for both images

5. CONCLUSION

In this paper various methods used for single image SR based on sparse representation and dictionary learning. For enhancing the scanning time of MRI images one effective way is to gain data from parallel multi-channel coils. Considering pMRI as a preprocessing step of MRI imaging helped us to perform DL on complex double data. Therefore, the scanning process is less painful for patients. The algorithm employed a mathematical framework of sparse representation and learning a coupled sparse DL on low resolution MRI and satellite images. To achieve this goal low rank sparse constraints and incorporating ADMM algorithm performed. Experimental results present that the proposed method is capable of SR for both satellite image and MRI imaging and it has a high accuracy in reconstruction process. Wide ranging experiments on image super-resolution confirm that by using Coupled dictionary learning and low rank constraints the proposed method obtains much better results than many state-of-the-art algorithms in terms of both PSNR, SSIM and visual understanding.

REFRENCES

Ayas,S., Ekinci, M.,2020. Single image super resolution using dictionary learning and sparse coding with multi-scale and multi-directional Gabor feature representation., *Information Sinces*.,512:1264-1278.

Barman,T., Deka,B.,Prasad,A.,2021. GPU-Accelerated Adaptive Dictionary Learning and Sparse Representations for Multispectral Image Super-resolution., *IEEE 18th India Council International Conference (INDICON)*.

Beck, A., Teboulle, M.,2009. Fast Gradient-Based Algorithms for Constrained Total Variation Image Denoising and Deblurring Problems., *IEEE TRANSACTIONS ON IMAGE PROCESSING*.,18(11) 2419-2434.

Boyd S., Parikh N., Chu E., 2011.Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends in Machine Learning*, 3(1): 1-122

Cevher V, Sankaranarayanan A, Duarte M. F, Reddy D, Baraniuk R. G, Chellappa R, 2008. Compressive sensing for background subtraction, *ECCV*, 155–168.

Chen, CH., Li, Y.,Huang, J., 2013.Calibrationless parallel MRI with joint total variation regularization., *Med Image Comput Comput Assist Interv*.,16:106-120.

Donoho, D.L.,2006. Compressed sensing. *IEEE Transaction on Information Theory*,52(4):1289-1306.

Donga,W., , Zhang, L.,2011. Image Deblurring and Super-resolution by Adaptive Sparse Domain Selection and Adaptive Regularization., *IEEE Transactions on Image Processing*.,20(7) 1838 – 1857.

Egiazarian K, Foi A, Katkovnik V,2007. Compressed Sensing Image Reconstruction Via Recursive Spatially Adaptive Filtering, *IEEE International Conference on Image Processing*, 552-549.

Fotiadou, K., Tsagkatakis,G., Tsakalides, P.,2019. Spectral Super Resolution of Hyperspectral Images via Coupled Dictionary Learning., *IEEE Transactions on Geoscience and Remote Sensing*, 57(5) 2777-2797

Gong, W., Han, SH., High-resolution far-field ghost imaging via sparsity constraint. *Scientific Reports*,9280(2015).

Hyvarinen, P., Hoyer, O., Inki, M.,2001. Topographic independent component analysis. *Neural computation*,13(7): 1527-1558.

Huang, D.T., Huang, W.Q., Huang, H., 2017.Application of regularization technique in image super resolution algorithm via sparse representation. *Optoelectron.lett*.13:439-443.

Jiao, Y., Jin, Q., Lu, X., Wang, W.,2016. Alternating direction method of multipliers for linear inverse problems, *arXiv* preprint arXiv:1601.02773.

Liu, J., Musialski, P., Wonka, P.,2013. Tensor Completion for Estimating Missing Values in Visual Data., *IEEE*

Transactions on Pattern Analysis and Machine Intelligence.,35(1) 208-220.

Marial, J., Jenatton, R., Obozinski, G., Bach, F., 2011.Convex and network flow optimization for structured sparsity. *Journal of* Machine *Learning Research*,12:2681-2720.

Merola, G.M., SPCA: sparse principal component analysis., 2014.*Pattern Recognit.Lett*.,34:1073-1045.

Sahebi R, Mojarradi B.,2018. A sliding window-based joint sparse representation (SWJSR) method for hyperspectral anomaly detection, *Journal of Remote Sensing* ,10:434-464.

Szlam, A, G., LeCun, Y.,2011. Structured sparse coding via lateral inhibition. In *Advances In Neural Information Processing Systems* (NIPS).

Zeyde, R., Elad, M., Protter, M.,2011. On Single Image Scale-Up Using Sparse-Representations., *Curves and Surfaces Springer (LNCS)*6920:711-730.

Zhang, H., Zhang, Y., Li, H.,2012. Generative Bayesian Image Super Resolution with Natural Image Prior., *IEEE TRANSACTIONS ON IMAGE PROCESSING,* VOL. 21(9) 4054-4067.

Zhang, H., Zhang, Y.,2009. Sparse representation based iterative incremental image deblurring., *16th IEEE International Conference on Image Processing (ICIP)*.

Zhou, H., Chen, L., Sapiro, G., Carin, L., 2009.Non parametric Bayesian dictionary learning for sparse image representations. *In Advances in neural information processing systems (NIPS*).

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https://doi.org/10.5194/isprs-annals-X-4-W1-2022-661-2023