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A THEORETICAL PATH COMPUTATION MODEL FOR RECTANGULAR RIGID-BODIES ENTERING AN ENTRANCE

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KEY WORDS: Path Computation, Entrance, Rigid-body in Rectangle-shape, User Dimensions, Navigation, Theoretical Model.

ABSTRACT:

Path planning is to calculate a path for users from the departure location to the destination by considering some conditions, such as travel distance, time, or safety. Although users are generally rigid-bodies with certain dimensions (e.g., cars, wheelchair), they are abstracted as a 0D point in current path planning. Such practice can simplify and accelerate path planning in environments where the spaces are large enough to ignore the users' dimensions. However, in many cases, the dimension of a user for path planning cannot be ignored, especially in some spaces where safety is critical, such as garage entrances. To help rigid-bodies safely enter into entrances, this paper presents a theoretical path computation model, in which users are rectangular rigid-bodies (RRB). This theoretical model can help the RRB-like users (e.g., robots, unmanned vehicles) to determine if an entrance is passable and further how to enter an entrance safely.

1. INTRODUCTION

Path planning is a process to find a collision-free path from departure to destination in an environment with obstacles according to certain criteria (Cai et al., 2008). In current research, during this process, the users are usually modelled as 0D points although they have certain dimensions, such as vehicles in outdoors (Jiménez et al., 2022), pedestrians and wheelchairs in indoors (Liu and Zlatanova, 2015). This can be partially explained that such practice can simplify and accelerate path planning when the spaces for navigation are spacious enough and compared to that of the spaces, dimensions of users can be ignored (Mac et al., 2017). Another possible reason is that the users are human beings or operated by people. For instance, pedestrians and vehicles in a navigation system are modelled as 0D points (Qiao et al., 2020), but they can adjust themselves or be precisely controlled at certain places.

The practice of abstracting users as 0D points is no longer workable in the development of unmanned systems, because users in such systems require strict control instructions for travelling, which are extremely important in places where their dimensions of them are comparable with those of the obstacles (Saboori et al., 2006), such as entrances. For an unmanned vehicle, if a path at an entrance is planned by using the vehicle as a 0D point, it may suffer a collision risk. Therefore, it is necessary to investigate a model that can consider the dimensions of users in path computation of entering entrances.

A common approach for this case is to model a rigid-body as its smallest circumcircle (Liu and Zlatanova, 2015), which could bring in some improvements, but it is inaccurate to rectangular rigid-bodies. For instance, modelling the vehicle in Figure 1 as a rectangular rigid-bodies is more precise than modelling it as its smallest circumcircle, since the area A and B are unnecessary for a path computation but they have decisive effects on determining whether an entrance is passable, i.e., if the radius of the smallest circumcircle is larger than the width of an entrance, the entrance will be determined as impassable. However, with proper control methods, the entrance is still passable, as long as the width of the entrance is larger than that of the vehicle.

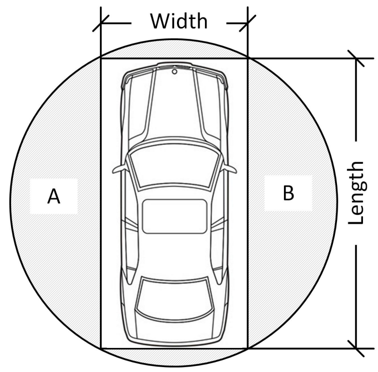

Figure 1. Comparison of modelling a rigid-body as its smallest circumcircle and a rectangle with length and width.

In this paper, to better describe the size characteristics of users, they are modelled as rectangular rigid-bodies (RRB) with length and width. Moreover, we deduce a theoretical model for an RRB-like user to help judging whether the entrance is passable and how to safely enter an entrance. The remainder of the paper is organized as follows. After exploring the related work in Section 2, we introduce the classification and modelling of different entrances and the factors affecting the path computation in Section 3. In Section 4, the theoretical model is introduced. In the last section, we make a summary and present future work.

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2. RELATED WORK

When it comes to the research of path planning that considers user dimension, the large majority of available literature focus on the influence of obstacles with different sizes and shapes, such as the hierarchical global path planning approach for mobile robots presented by (Mac et al., 2017).

In the research of path planning with respect to the user dimension (Liu and Zlatanova, 2015), the authors considered presented a new approach, in which the users (a person and her/his operated objects) are modelled as circles. Then, radius of the circle is utilized to determine inaccessible gaps. In particular, finding the unreachable area by comparing the radius with the smallest distance between obstacles. After identifying groups of obstacles, they computed boundaries for the selected group. Finally, a network in the accessible area around the obstacles is built for path planning. However, this research only presents a common way to determine if some openings (gaps) are accessible, no more details about entering the such openings. Moreover, as mentioned in Section 1, modelling a user as a circle is inaccurate.

Another typical research that considers users dimension is the path planning for carrier-based aircraft on the surface of the aircraft carrier (Zhang et al., 2014b, Zhang et al., 2015, Wang et al., 2021). The aircraft on decker are rigid bodies with complex shapes and they have very strict requirements for the collision-free path. Thus, they cannot be modelled as 0D points. In (Zhu et al., 2021, Zhang et al., 2014b, Zhang et al., 2014a), to reduce the complexity of the model but without losing effective spaces, a polygon line segment set is introduced to describe a carrier-based aircraft. Similarly, polygon bounding box is another way to model the carrier-based aircraft (Wang et al., 2021). Yet, such research also only using the dimension model to determine the distance between users and obstacles in the subsequent path planning.

Other than that, the dimension of users is investigated in the path planning of Automatic Car Parking Systems (ACPS). There are several ways to generate the parking trajectory considering the user dimension. For instance, (Sungwoo et al., 2011) presented a geometry-based approach that takes the geometry of a vehicle and its maximum steering angle as the references to determine the minimum turning radius. Further, (Liang et al., 2012) added Bezier curve fitting to make the trajectory of the car more smooth for parking. In the research of ACPS, on the basis of the geometric characteristics of cars, researchers can propose different methods to improve the automatic parking process.

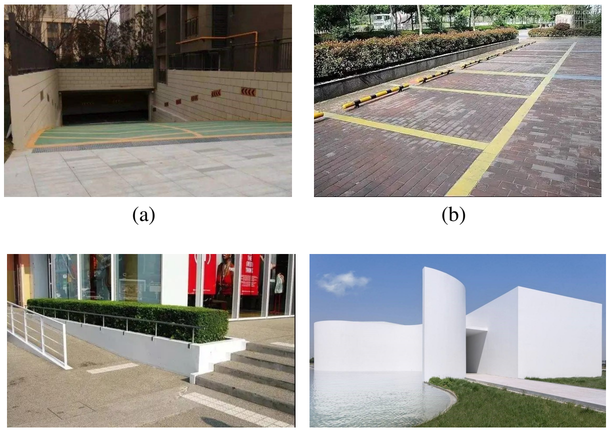
Another slightly similar but different to our research topic is ‘moving a couch around a corner’ (Moretti, 2002) or the more general research that ‘moving a rectangle around a corner’ (Boute, 2004). Such topics focus on determining if a (rectangle) rigid body, such as sofa, ladder, can pass through a certain corner by rotations. As for the rotating movements in the corner space, they are operated by humans. Such research cannot handle the issue that if a RRB can enter and how to enter an entrance safely. After entering the entrance, some corners may be there. Then, the topic becomes moving a rectangle around a corner.

In short, current research only focuses on finding a collision-free path but does not pay attention to path computation of the rigid bodies when they have to pass through some openings. In this paper, we not only model the RRB but also presents a theoretical model for computing the trajectories in the process of entering an entrance.

3. MODELLING OF ENTRANCE AND USER

3.1 Modelling of Entrance

An entrance is an opening structure, such as a door, passage, or gate, that allows access to a space. The shape of entrances can be various (Figure 2). Figure 2(a) is a common underground garage entrance. The length of boundaries on both sides of this entrance is the same. Figure 2(b) are parking spaces, which also have entrances but their boundaries are virtual. Figure 2(c) and Figure 2(d) are also two entrances, but the length of the boundaries that form the entrances are different.



(a) (b) (c) (d)

Figure 2. Examples of different types of entrances.

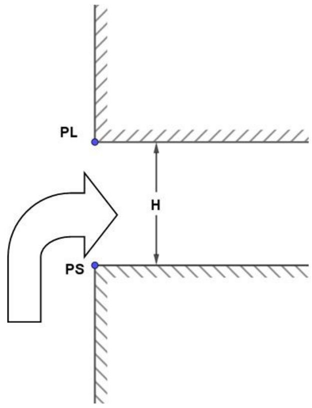


Figure 3. Modelling of entrance.

The most common entrance in our life is that the length of the boundaries on both sides is equal, such as doors, tunnels, etc. Thus, this paper focuses on this type of entrance. To simplify such kinds of entrances, we modelled them as opening structures that are formed by two symmetrical boundaries from up and down (Figure 3). In the figure, the arrow shows the movement direction of an RRB. This means that the purpose of this paper is to calculate the path that can help an RRB enter the entrance when it travels forward. Although if the space in front of the entrance is enough spacious, the RRB may adjust its direction to enter the entrance directly, this case is not within the scope of this paper.

3.2 Modelling of an RRB

As mentioned in the introduction, the users in this research are rigid-bodies, and they are further modelled as rectangular rigid-bodies (RRB) with length and width (Figure 4). In order to facilitate subsequent calculations, the length of the RRB is set as $2L$ while its width is W. The arrow on the RRB show its forward direction. The third dimension - height is ignored in this paper, as it has no direct effect on the path computation.

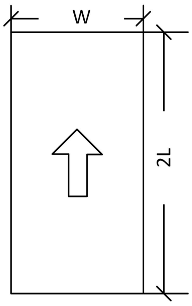


Figure 4. Illustration of an RRB.

For the entrance shown in Figure 3, an RRB needs to turn/rotate to enter an entrance. If the motion involves both translation and rotation, there will be a temporary static point on the rigid body, which can be called as instantaneous centre of rotation (ICR) (Boute, 2004). Theoretically, ICR can be located anywhere on the RRB. In this paper, the ICR is determined as the midpoint of its right boundary. We mark ICR as C here. Figure 5 shows trajectory and required space of RRB, in which ICR is the midpoint of its right boundary.

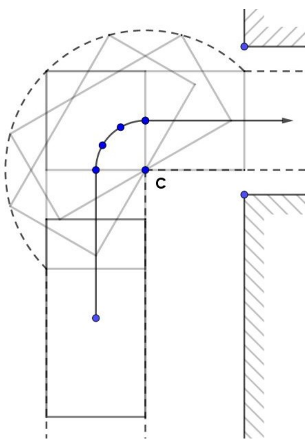


Figure 5. The trajectory and required space of RRB with a rotation centre. The line with the centre of RRB is the trajectory and the space between the two dash lines is the required space.

4. THEORETICAL PATH COMPUTATION MODEL

4.1 Overview of the Model

The presented theoretical model aims to compute the trajectories in the process of entering an entrance. The first step is judging whether an entrance is accessible by comparing the width of RRB (W) with that of the entrance (H). Then, the rotation centre is set, which has been shown in Figure 5. As the lengths of up and down boundaries of the entrance are equal, RRB can enter it in the same way from the opposite direction.

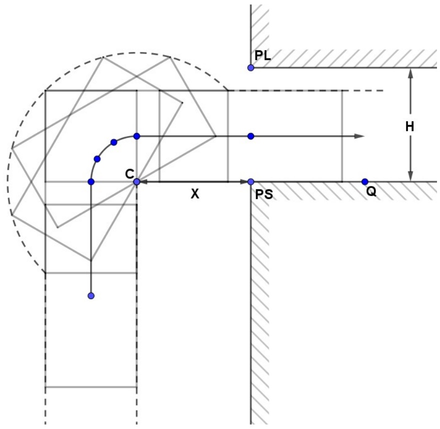


Figure 6. The illustration of the theoretical path computation model for an RRB.

The overview of the model is shown in the Figure 6. The trajectory of the RRB is that its rotation centre will move to a location and have a rotation, in which the location is the intersection of the right boundary of the RRB and the extension of the line segment formed by Q and PS.

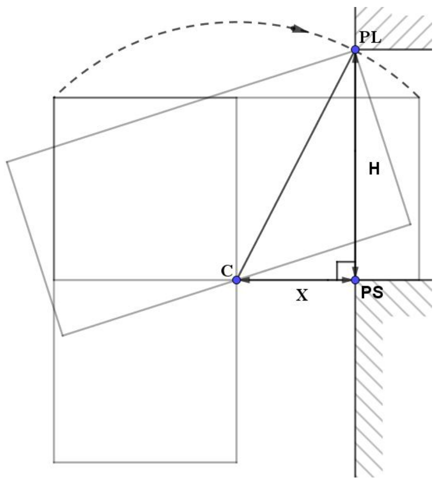


Figure 7. determination of theoretical limit value.

As shown in the Figure 7, to make sure the RRB enter the entrance safely, there is a minimum distance between the C and the corners of the entrances. In particular, we mark that distances as $L_{PL,C}$ and $L_{PS,C}$. Since the $L_{PS,C}$ is variable, we future mark it as X. The X in this figure is the shortest theoretical distance. According to Pythagorean theorem, the value of X is:

$$X = \sqrt{L_{PL,C}^2 - L_{PL,PS}^2} = \sqrt{L^2 + W^2 - H^2} \qquad (1)$$

where $L_{PL,C}$ = the distance PL and C
$L_{PL,PS}$ = the distance between PL and PS
L = half the length of RRB
W = width of RRB
H = width of entrance

According to our observations, we found that the trajectory and required space of an RRB will be different when X

varies from 0 to infinity. Thus, we split it as three cases: $X = 0$ (Case I), $X \geq \sqrt{L^2 + W^2 - H^2}$ (Case II), and $0 < X < \sqrt{L^2 + W^2 - H^2}$ (Case III).

4.2 Case I: $X = 0$

The $X = 0$ means that the right boundary of RRB is touching the boundary of the entrance. For this case, to make sure the RRB can enter the entrance safely, the H should larger than $\sqrt{L^2 + W^2}$. Then, the trajectory and required space for this RRB can be determined (Figure 8).

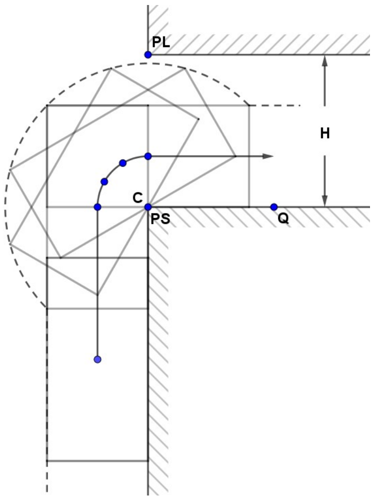


Figure 8. The trajectory of RRB when $X = 0$.

4.3 Case II: $X \geq \sqrt{L^2 + W^2 - H^2}$

If the RRB keeps a distance with the boundary of entrance, i.e., $X! = 0$, the location for rotation will change. When the RRB is able to enter the entrance by rotation, the minimum X is $\sqrt{L^2 + W^2 - H^2}$. In this case, the location for rotation (T) is located at (i) the extension of the line segment formed by Q and PS, and (ii) the distance between T and PS is X (Figure 9). During the entering process, the RRB begins to rotate when the C overlaps to T.

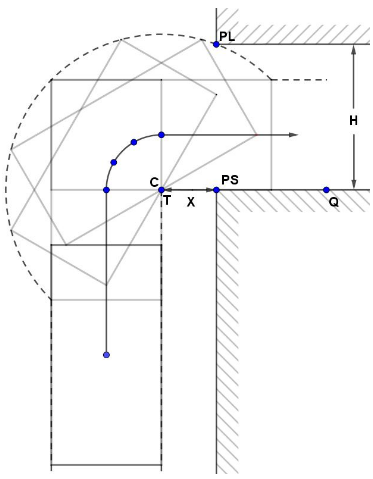
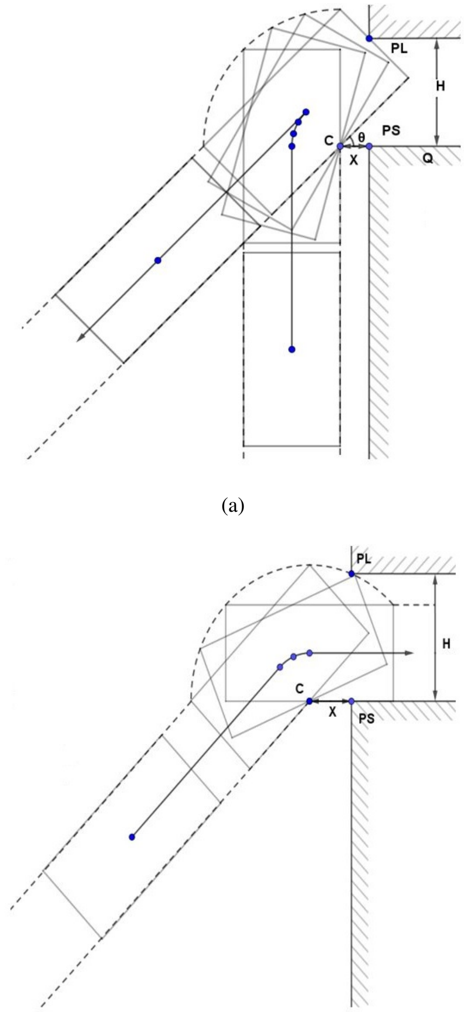


Figure 9. The trajectory of RRB when $X \geq \sqrt{L^2 + W^2 - H^2}$.

4.4 Case III: $0 < X < \sqrt{L^2 + W^2 - H^2}$

Another case is $0 < X < \sqrt{L^2 + W^2 - H^2}$. It means that there is a distance between RRB and the entrance, and this distance is too close for the RRB to enter the entrance safely by a rotation. Then, the entering process includes three stages. In the first stage, the movements of RRB are similar to the other two cases, but the rotations will be stopped until the boundary of RRB is touching the PL. As shown in the Figure 10(a), the rotation angle is θ. In the second stage, the RRB will move a distance away from the entrance. Finally, the RRB will move forward until the C overlaps to T. Then, the rest process is exactly the same as the case II. The whole process can be seen in Figure 10(b).



(a)

(b)

Figure 10. The trajectory of RRB when $0 < X < \sqrt{L^2 + W^2 - H^2}$.

Figure 11 shows the rotation angle (θ) and the shortest moving distance (S) of RRB. Because $\varphi + \theta = 90^\circ$ and $\angle ACPL = \angle\varphi$, $\theta = 90^\circ - \varphi = 90^\circ - \angle ACPL$. And $\angle ACPL$ can be calculated by law of cosines. In ΔACPL, $L_{C,PL} = L_{A,C} = \sqrt{X^2 + H^2}$, $L_{A,PL} = \sqrt{(L_{A,B} + X)^2 + (H - L_{B,C})^2}$. Therefore, the θ is:

$$\theta = 90^\circ - arccos\frac{HL - X\sqrt{X^2 + H^2 - L^2}}{X^2 + H^2} \tag{2}$$

where L = half the length of RRB
H = width of entrance

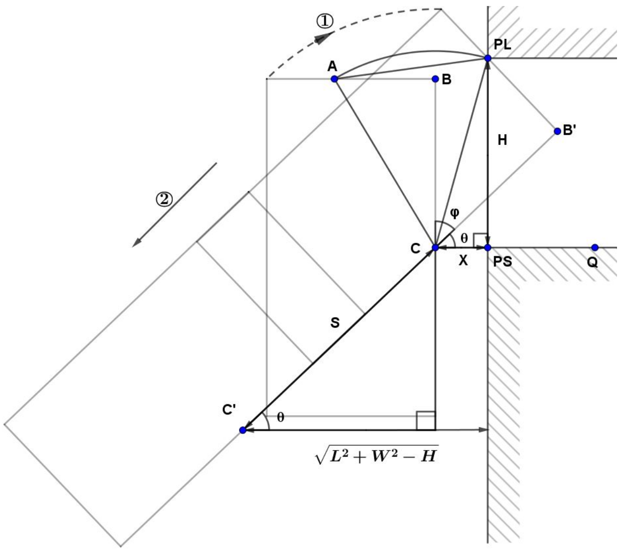


Figure 11. The illustration of calculation process of θ.

X = the distance between C and PS.

In the equation 2, L, H and X are known. When the distance(X) between RRB and entrance is too close, the included angle between RRB and entrance becomes constant. Thus, the rotation angle of RRB can be computed based on the included angle between $\overrightarrow{CB}$ and $\overrightarrow{CPS}$ minus θ. Having θ, the minimum distance (S) can be calculated by Equation 3:

$$S = \frac{\sqrt{L^2 + W^2 - H^2} - X}{cos\theta} \tag{3}$$

where X = distance between C and PS
θ = angle between RRB and entrance

4.5 THE WHOLE PROCESS OF RRB ENTERING THE ENTRANCE

Previously, we described the theoretical parts required by the theoretical model and the path of RRB in three different cases. This section describes the whole process of RRB entering the entrance (Figure 12). The movement of RRB in the whole process is divided into four parts: rotation, moving, re-rotation and move forward. As we can see from the above sections, the starting position of RRB, the initial value of C, T, PL and PS are known. As for the other end points of RRB (B, B', B'', etc.), we can get them according to C and their geometric relationships.

4.6 DISCUSSION

The entrance in this research is only one common type of entrance. For other types of entrances, such as those shown in Figure 2(c) and 2(d), something of the presented theoretical model should be adjusted, since if we still model an entrance as an opening structure that formed by two boundaries from up and down, the two boundaries could have different length.

Having proper approach for modelling rigid-bodies is critical for path computation. Theoretically, the more accurate the

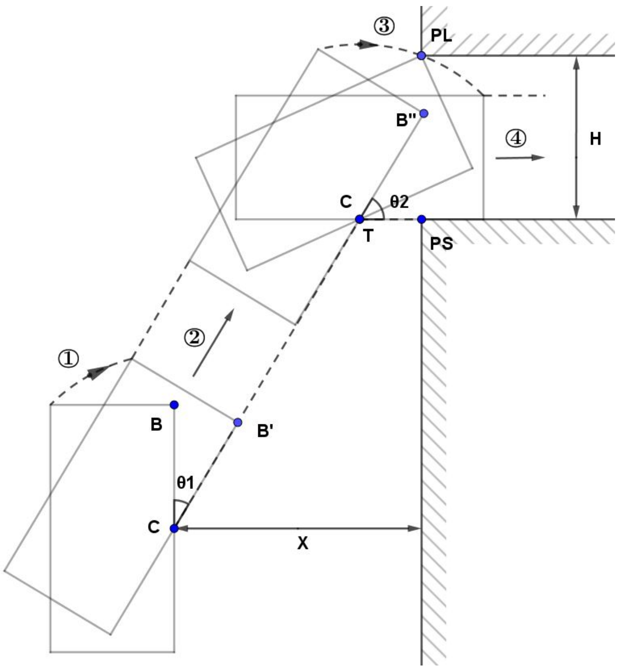


Figure 12. overall path of RRB entering the entrance.

modelling of the user dimension, the more realistic planned path, and the safer the navigation will be, but obviously, the computational complexity of path planning will inevitably increase. For instance, using the actual car directly as a model for path planning (Ye et al., 2019) is the most precise way, but it is not the recommended. This paper models users as RRB, which may be too rough to deal with some situations.

In this research, the rotation centre is the midpoint of an RRB' right boundary. However, it may locate differently, which will have influence on the trajectories and the required spaces. For instance, Figure 13 shows other two types of rotation centre. The rotation centre of the Figure 13(a) and 13(b) is the centre of the RRB, while that of the Figure 13(c) is the vertex in the upper right corner of the RRB.

5. CONCLUSION AND FUTURE WORK

In this paper, we propose a theoretical model to help rectangular rigid-bodies (RRB) to enter an entrance, in which the upper and lower boundaries of the entrance are flush and symmetrical. In the path computation, the dimension of RRB is considered. This research is critical for path planning of unmanned systems, such as unmanned ground vehicle (UGV), autonomous vehicles. The presented model only depends on the dimension of RRB and entrance, thus, it is robust and not affected by other factors.

In future work, we will concentrate on further elaboration and testing of the current theoretical model as follows: (i) model and investigate the path computation for the non-flush and unsymmetrical entrances; (ii) take the users with other shapes into consideration; (iii) conduct simulations and tests; (iv) integrate the presented model into navigation systems.

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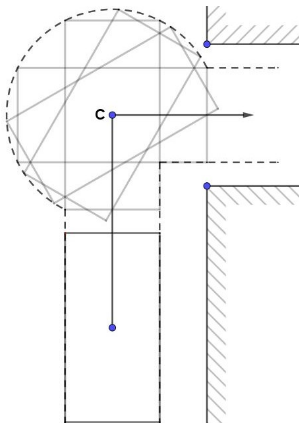
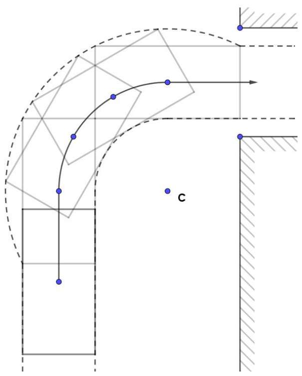


(a)



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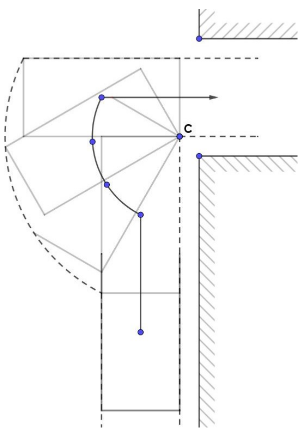

(c)

Figure 13. The trajectories of RRB with different rotation centres. All the rotation centres are C.

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