

A Density-Based Clustering Method and Beam Search for a Capacitated Vehicle Routing Problem with Time Windows

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ABSTRACT:

This study introduces a multi-stage strategy for addressing the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) in the context of Iran's national postal system. The proposed method integrates a density-based spatiotemporal clustering technique with a heuristic optimization process to enhance the efficiency of large-scale logistics management. To reduce computational complexity, the MDST-DBSCAN algorithm is utilized to cluster provincial postal centers based on operational and spatial characteristics. Integrating operational and spatial features of problem in clustering process, brings the CVRPTW closer to real-world. Once clustering is complete, a beam search heuristic is applied to create feasible delivery routes within each cluster. The beam search algorithm identifies optimal routes by searching multiple route in parallel. These routes are then fine-tuned using a 2-opt local search technique. Overall, the framework effectively simplifies the complexity of route planning and aligns with real-world postal logistics needs, achieving notable improvements in both route length reduction and fleet usage.

1. INTRODUCTION

The surge in e-commerce activity has led to a continuous increase in the volume of parcels handled by postal services. While traditional letter mail continues to decline, package shipments have seen steady growth. One of the major challenges facing modern postal logistics is the need to deliver parcels within tight time windows using a limited number of vehicles. Studies show that for e-commerce customers, timely and dependable delivery is the highest priority. However, generating optimal delivery routes under real-world constraints—particularly time windows—renders the problem NP-Hard. As noted by Žunić et al. (2022), effective transportation systems have the potential to increase efficiency, lower costs, and improve service quality. Although traditional routing techniques have long been used in postal systems, they are not well-suited for obtaining exact solutions in large-scale scenarios (Dornemann et al., 2023). This research models the routing challenge across Iran's national postal network as a Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). The task involves a fleet of vehicles—each with a fixed capacity—assigned to deliver parcels to postal centers located in all 31 provinces, each with specific time windows and distributed across a broad geographic area. To address this, a three-phase algorithmic framework was developed. In the first phase, the MDST-DBSCAN algorithm was used to cluster provincial postal centers based on spatial positioning, parcel volume, and the availability of vehicles, thereby reducing computational complexity. This clustering method effectively identifies geographically and operationally coherent zones (Choi & Hong, 2021). In the second phase, the

Beam Search algorithm is used to generate initial delivery routes within each cluster, ensuring compliance with both capacity limits and delivery time constraints. By utilizing guided search, this method produces high-quality, scalable solutions for logistics systems operating under strict time windows (Tang et al., 2014).

2. RELATED WORK

The Vehicle Routing Problem (VRP), introduced by Dantzig and Ramser in 1959, is a core issue in combinatorial optimization and logistics. It involves assigning a fleet of vehicles to deliver goods from a central depot to a set of geographically dispersed customers, aiming to minimize overall travel distance or time (Dantzig & Ramser, 1959). A common extension of this problem is the Capacitated Vehicle Routing Problem (CVRP), which includes vehicle capacity limits. A further variation, the Vehicle Routing Problem with Time Windows (VRPTW), introduces time constraints for customer service. When both of these conditions—vehicle capacity and time windows—are combined, the problem becomes the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). This model is particularly relevant to industries such as express delivery and postal logistics, where both timeliness and capacity efficiency are essential. One prevalent strategy for solving CVRPTW involves decomposing the main problem into smaller, more manageable sub-problems using clustering techniques. Clustering for VRP typically considers factors such as customer location, service time windows, and quantity constraints (Žunić et al., 2022). Among these clustering methods, the K-means algorithm and its

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variations have been widely applied to group customers or demand points (Geetha et al., 2009; Ha & Moon, 2015; Mostafa & Eltawil, 2017; Revanna et al., 2022; Zhang et al., 2025). Additionally, density-based approaches like DBSCAN and its derivatives have proven useful in various routing problem contexts (Bujel et al., 2018; Gocken & Yaktubay, 2019; Villalba & Rotta, 2020; Lin et al., 2024). Beyond these, other methods such as hierarchical clustering (Liu et al., 2015), spectral clustering (Chen et al., 2017), self-organizing maps (Fritz et al., 2002), genetic algorithm-based clustering (Prins, 2004), fuzzy clustering (Kuo et al., 2008), and graph-theoretical techniques (Delling et al., 2013) have been used to enhance customer segmentation based on location and demand. However, many of these methods lack the ability to fully capture spatiotemporal patterns, especially in the context of complex delivery networks. To address this gap, Birant and Kut (2007) proposed the ST-DBSCAN algorithm, which incorporates both spatial closeness and temporal overlap in clustering. Later, the MDST-DBSCAN algorithm was introduced by Choi and Hong (2021) to allow clustering based on multiple time-sensitive and operational features, such as demand volume, vehicle availability, and parcel size. This method is particularly well-suited to large, dynamic postal networks where time, location, and workload are interconnected. The ability to adapt routing plans to fluctuating demand—shaped by seasonality, local events, or policy changes—makes this approach highly practical in real-world logistics. Ongoing research on VRP has produced a wide range of problem-solving strategies. In general, these methods fall into two categories: exact algorithms, which are limited to smaller instances due to their computational intensity (Desrosiers et al., 1995), and more scalable exact approaches that push those limits (El-Sherbeny, 2010). For large-scale problems, heuristic and metaheuristic algorithms are widely adopted. A survey by Dixit et al. (2018) explored several of these, including Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), and Artificial Bee Colony (ABC). For example, Lim and Zhang (2007) developed a two-stage heuristic incorporating “Ejection Pools” for solving VRPTW. Chao et al. (1996) proposed a fast heuristic for the orienteering problem, where each customer provides a profit value and the goal is to maximize total profit. Goel and Maini (2018) presented a hybrid approach combining ACO and the Firefly Algorithm (FA). Among heuristic methods, beam search has received comparatively little attention in the VRP literature. Originally proposed by Lowerre (1976), beam search is a tree-based strategy that explores only the most promising options at each level, rather than performing a full breadth-first search. By limiting the number of branches it expands, it remains computationally tractable, making it an appropriate choice for VRPs with time window constraints (Zhang et al., 2025). Akebi et al. (2013) applied beam search in a three-stage method for CVRPTW, beginning with K-means clustering, followed by route generation using beam search, and final refinement via local search. Their findings showed that combining beam search with decomposition strategies can yield strong results. Nazari et al. (2018) used beam search in a hybrid machine learning framework to decode routes generated by pointer networks, emphasizing its role in achieving high-quality solutions even in learning-based models. Though less explored than other heuristics, beam search offers valuable advantages. It reduces the search space efficiently, supports parallelization, and scales well with the demands of real-world vehicle routing tasks.

3. METHODOLOGY

3.1 Study Area and Data set

Iran’s postal system spans all 31 provinces and operates across an extensive geographic area through a centralized network of six regional postal hubs (Tehran, East Azerbaijan, Razavi Khorasan, Fars, Hamedan, and Isfahan). The dataset used in this study consists of detailed operational records collected daily from January 2022 to December 2023. For each province, the data includes the number and weight of both incoming and outgoing parcels, the volume of vehicular traffic (arrivals and departures), and metrics for each delivery route such as travel time, route distance, and any recorded delays or early arrivals. Figure 1 displays the geographic distribution of the provincial postal centers, the locations of the six main hubs, and the interprovincial postal routes.

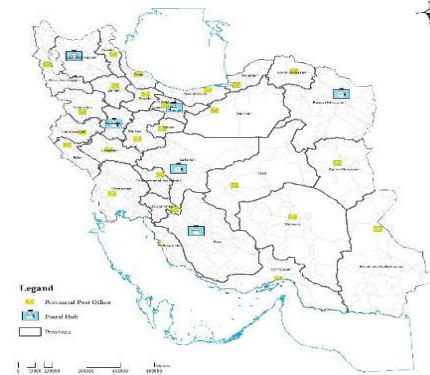


Figure 1. Spatial positing of Provincial postal center

3.2 Mathematical formulation CVRPTW

Each customer $i \in N = \{1, 2, \dots, n\}$ is defined by their coordinates (x_i, y_i) in a two-dimensional Euclidean space. The delivery fleet consists of m vehicles $V = \{v_1, \dots, v_m\}$, each with a fixed carrying capacity Q_{max} . All vehicles originate from a central depot Hub D , located at coordinates (x_D, y_D) , and are required to return to the same hub after completing their delivery routes. Each customer $i \in N$ places a specific demand, typically expressed as parcel weight or volume, and must be serviced within a predefined time window $W_i = [a_i, b_i]$. Each vehicle $v_i, 1 \leq j \leq m$, is assigned a route R_j that includes a subset of customers within a given cluster C_j . The total travel distance for vehicle v_j is denoted by D_j . The Euclidean distance between any two customers i and j is denoted as $dist_{i,j}$. The primary objective is to minimize the number of vehicles m and the total distance traveled $\min \sum D_i, 1 \leq j \leq m$, subject to the following constraints:

- The total demand for each cluster C_j must not exceed the vehicle capacity Q_{max} : $\sum_{i \in C_j} d_i \leq Q_{max}, \forall i \in C_j$
- Each customer i must be served within their specified time window $[a_i, b_i]$: $t_i \in W_i = [a_i, b_i] \forall i \in N \cup \{D\}$
- The hub serves as both the starting and ending point for each vehicle route.

3.3 CVRPTW Clustering with MDST-DBSCAN

To improve computational scalability and reduces overall complexity, the 31 provincial postal centers are divided into 6 distinct clusters. Density-based clustering more suitable when dealing with irregular spatial distributions of postal centers, as they can identify clusters of arbitrary shapes. The DBSCAN

algorithm introduced by Ester et al. in 1996, which performs clustering using the Eps and MinPts parameters. To support spatiotemporal data, ST-DBSCAN (Birant & Kut, 2007) introduced two spatial and temporal thresholds. Subsequently, MDST-DBSCAN (Choi & Hong, 2021) extended this method by adding temporal and operational dimensions such as demand volume and vehicle availability. In our work, we adopt MDST-DBSCAN as the clustering method, because it is capable of handling multiple temporal and operational features. A spatio-temporal feature space was defined for each postal center including geographic coordinates, volume and weight of inbound and outbound parcels, and number of vehicles. The algorithm operates by randomly selecting an unvisited point in the dataset and evaluating all nearby points using the defined Eps thresholds to determine whether they form a cluster. The following thresholds were configured for MDST-DBSCAN: Euclidean spatial distance (Eps_1), number of vehicles (Eps_2), number of postal parcels (Eps_3), and weight of postal parcels (Eps_4). For two provincial postal centers, $A(x_1, y_1, t_1, t_2)$ and $B(x_2, y_2, t_3, t_4)$, Eps_1 and Eps_2 are calculated as:

$$Eps_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (1)$$

$$Eps_2 = \sqrt{(v_{t_1} - v_{t_2})^2 + (v_{t_3} - v_{t_4})^2} \quad (2)$$

Eps_3 and Eps_4 are calculated in the same manner as Eps_2 . To form a valid cluster, all four Eps thresholds must be satisfied simultaneously, and the number of neighboring points must exceed the *MinPts* parameter. This process continues iteratively until all points in the dataset are evaluated. To ensure a balanced distribution of clusters, a minimum number of clusters and a minimum number of postal centers were considered, which leads to a more balanced clustering consistent with the real-world postal delivery system.

3.4 Finding the shortest path with beam search:

After identifying the optimal spatio-temporal clusters, the second stage of the process focuses on finding the shortest route within each cluster. Each route must start from its assigned postal hub, visit every postal center in the cluster exactly once, and meet all problem constraints, including time windows and vehicle capacity. The goal in this stage of the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) is to minimize the total travel distance traveled by the m vehicles within each cluster. For this purpose, Beam Search, an optimized version of the Branch and Bound algorithm, is a suitable choice. The search process in **beam search** begins by creating a **root node** (Lowerre, 1976), which may be a **partial solution**. In the next step, each node at level l generates a set of **descendants** corresponding to level $l + 1$. The nodes created at the new level are evaluated using an **evaluation criterion**, and only a **subset of the best nodes**, ω (**beam width**), are retained while the others are discarded. If a node contains a **final solution**, it is then **evaluated and stored**. That node is subsequently **removed**, since no branch (leaf) from it is feasible (Lowerre, 1976). Beam search stops when **no branch from any node at the current level** is feasible. The **best solution** among the various obtained solutions is then retained as the **final result**. If a **node** represents a **complete (final) solution**, it is **evaluated and stored**. Since no further expansion of such a node is possible, it is treated as a **leaf** and **removed from the search tree**. The **beam search** continues this process until **no branching from any node at the current level** is possible. Once the search terminates, the **best solution** among all evaluated candidates is selected as the **final result**. Let

$P = \{p_1, \dots, p_j\}$ be the set of postal centers in cluster C_j ($1 \leq j \leq m$). Each node η_l at level l contains the following elements:

- The set of postal centers (or customers) already visited $P^+ = \{p_1^+, \dots, p_l^+\}$.
- The set of postal centers (or customers) that have not yet been visited $P^- = P \setminus P^+$.
- Distance related to the length of the path (*dist*): $D \rightarrow p_1^+ \rightarrow p_2^+ \rightarrow \dots \rightarrow p_l^+$.

Note that if a node corresponds to a complete solution, then the path obtained represents a feasible route that satisfies all problem constraints and is subject to evaluation for optimality ($D \rightarrow p_1^+ \rightarrow p_2^+ \rightarrow \dots \rightarrow p_{|C_j|}^+ \rightarrow D$). The total distance is calculated as the sum of the Euclidean distances of the arcs that constitute the corresponding path. At final step, a node η_l at l level in the search tree can be formally characterized by the elements described above, i.e., $\eta_l = \{P^+, P^-, dist\}$, where $|P^+| = l$ and $|P^-| = |C_j| - l$.

Branching from a node involves the successful selection of vertex p_l^+ from among the vertices V^- . The next vertex $p_i \in P^-$ may be the closest to p_l^+ in terms of Euclidean distance or time window $[a_i, b_i]$. In this research, only the time window constraint is considered, because the capacity of each cluster is compatible with the capacity of the vehicle and the decision be made based on the distance between customers. The input parameters of beam search include the cluster C_i , beam width (ω) and selection criterion (p) for ranking nodes. In the proposed implementation of the algorithm, beam width is set to 10 and using heuristic pruning, only valid paths based on cost, time and capacity are retained.

3.5 Local search for improving solution quality

To enhance the solution obtained from Beam Search, we apply the well-known 2-opt heuristic algorithm (Barma et al., 2019) within each cluster. This algorithm iteratively improves a given route by removing two nonconsecutive edges and reconnecting the resulting segments in a way that reduces the total travel distance, while still respecting time windows and vehicle capacity constraints (Barma et al., 2019). In each iteration, the method selects two nonconsecutive arcs (edges) in the current route, removes them, and reconnects the segments in an alternative way to form a new feasible route.

4. COMPUTATIONAL RESULTS

The proposed algorithm was developed using Python 3.8 and runs on a Windows system with 2 GB of RAM and an Intel 3 GHz processor. Data preprocessing and editing were handled using popular libraries like Pandas and NumPy. For clustering the provincial postal centers, we used the scikit-learn library. The Beam Search method was implemented using Python's heapq module to efficiently manage the priority queue. The 2-opt algorithm was coded from scratch, with optional support from the tsplib95 library for problem formatting. For interpreting results and visualizing data, we used Matplotlib and Seaborn. The Beam Search solver generates optimized vehicle routes within each cluster, assigning groups of provinces to specific postal centers. The output includes the total distance traveled by all vehicles in each cluster and the total number of vehicles used. A summary of these results is provided for each postal hub, showing the number of routes, distances covered, and vehicles deployed.

Cluster	Postal Hub	Postal center province
0	Fars	Bushehr, Fars, Kohgiluyeh & Boyer-Ahmad, Yazd
1	Razavi-Khorasan	North Khorasan, Razavi Khorasan, South Khorasan, Semanan, Golestan, Mazandaran
2	Isfahn, Hamadan	Isfahn, Ilam, Qom, Kermanshah, Lorestan, Markazi, Hamedan, Khuzestan, Chaharmahal and Bakhtiari
3	Tehran, East Azerbaijan	East Azerbaijan, West Azerbaijan, Ardabil, Alborz, Tehran, Zanjan, Qazvin, Gilan, Kurdistan
4	-	Sistan and Baluchestan, Kerman, Hormozgan

Table 1. Province Clusters Identified by MDST-DBSCAN for Regional Hub

According to the Table 1, clustering results shows that some clusters have multiple regional centers and some, like the fourth cluster, have no centers, which indicates the need to use cluster aggregation or division techniques.

Cluster	Postal Hub	Postal center province
0	Fars	Bushehr, Fars, Kohgiluyeh & Boyer-Ahmad, Yazd, Sistan and Baluchestan, Kerman, Hormozgan
1	Razavi-Khorasan	North Khorasan, Razavi Khorasan, South Khorasan, Semanan, Golestan, Mazandaran
2	Isfahn	Isfahn, Qom, Markazi, Khuzestan, Chaharmahal and Bakhtiari
3	Hamedan	Hamadan, Lorestan, Kermanshah, Ilam
4	Tehran	Alborz, Tehran, Qazvin, Gilan, Zanjan
5	East Azerbaijan	East Azerbaijan, West Azerbaijan, Ardabil, Kurdistan

Table 2. Province Clusters after aggregation and division

By dividing the clusters, the provinces are assigned to their appropriate hubs such as East Azerbaijan, Tehran, Isfahan, and Hamedan, and clusters 4 and 0 are merged. K-means or DBSCAN algorithms is used for division, and cluster aggregation is done based on geographical distance. Table 2 illustrates the clustering results obtained using the MDST-DBSCAN algorithm across the 31 provinces of the country. The distribution of

clusters reflects both geographic proximity and temporal alignment of postal flow patterns. This spatiotemporal clustering supports demand forecasting and resource allocation by grouping provinces with similar demand characteristics, thereby aiding the development of more robust and scalable routing strategies. The MDST-DBSCAN algorithm results group the country's postal centers into five main clusters.

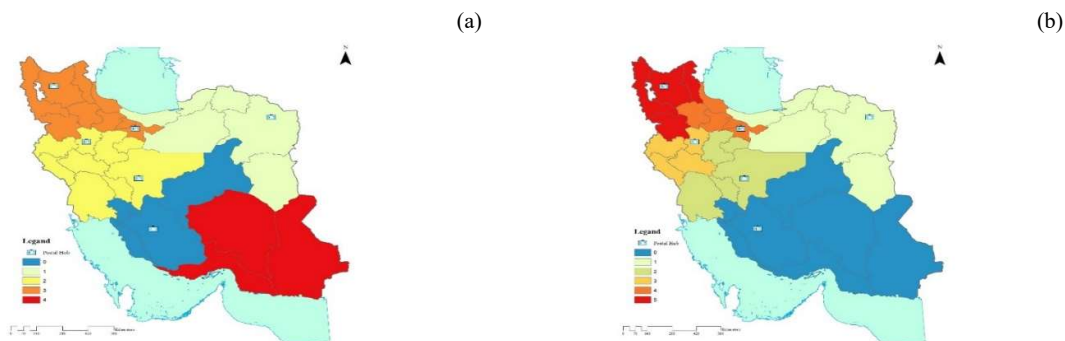


Figure 2. Spatial presentation of Provincial postal center from MDST-DBSCAN Clustering
 (a) Clusters Identified by MDST-DBSCAN, (b) Province Clusters after aggregation and division

Figure 2. Shows the spatial results of the MDST-DBSCAN algorithm after the clustering and aggregation process. After this configuration, each of the six hubs serves its closest neighboring provinces, which reduces time, and accelerates parcel delivery.

No of Cluster	No of postal center	Traveling distance (km)	ω_{max}	Solving time (Sec)
Tehran	5	638.2km	3	185
Fars	7	2953.6	4	98.16
Hamedan	4	605.23	3	25.44
Isfahan	5	1330.2	3	167.88
East Azerbaijan	4	935.81 km	3	57.44
Razavi Khorasan	6	2250	4	121.69

Table 3. Summary of Beam search result per postal Hub

Table 3 shows the results of the beam search algorithm for each cluster (or postal hub), including the optimal distance and the execution time of the algorithm. Figure 2 shows the optimal routes resulting from beam search. These results allow for effective spatio-temporal planning and vehicle resource allocation based on actual demand.

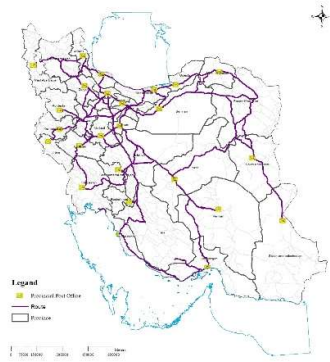


Figure 3. Optimal routes for delivery postal parcels

5. CONCLUSION AND FUTURE WORK

This study proposes a hybrid framework to address the CVRPTW problem using real data from Iran's postal network. Given the irregular spatial distribution of provincial postal centers, a density-based algorithm was employed for spatial-temporal clustering. The application of the MDST-DBSCAN algorithm to divide Iran's postal network into six spatiotemporal clusters offers an innovative approach to enhancing operational efficiency in logistics routing. By incorporating more than two temporal features alongside spatial attributes, the clustering process effectively captures the complex dynamics of parcel flows and delivery schedules. The clustering results facilitate key tasks such as forecasting the inflow and outflow at each postal hub and allocating vehicles based on these predictions, thereby simplifying the planning of parcel volume fluctuations during special occasions or peak periods. This clustering approach, which decomposes the problem into smaller subproblems, improves the algorithm's efficiency in solving the routing challenge and shows great potential for redesigning intelligent delivery systems. The core step in solving the CVRPTW problem involves computing the shortest routes within each cluster. For this, the heuristic Beam Search algorithm was employed. A notable advantage of this method is its ability to examine multiple routes in parallel, increasing the chances of identifying optimal paths. Future research could explore the use of adaptive beam widths instead of fixed ones during the search process, allowing more flexible exploration depths and branching, which may enhance solution diversity and robustness. Additionally, future studies should investigate the integration of real-time data, traffic conditions, and various machine learning and heuristic optimization techniques to develop innovative solutions for last-mile logistics challenges.

REFERENCES

Akeb, H., Bouchakhchoukha, A., Hifi, M., 2013: A beam search based algorithm for the capacitated vehicle routing problem with time windows. *IEEE, 2013 Federated Conference on Computer Science and Information Systems*, pp. 329–336.

Barma, P. S., Dutta, J., Mukherjee, A., 2019: A 2-opt guided discrete antlion optimization algorithm for multi-depot vehicle routing problem. *Decision Making: Applications in Management and Engineering*, 2(2), pp. 112–125.

Birant, D., Kut, A., 2007: ST-DBSCAN: An algorithm for clustering spatial-temporal data. *Data & Knowledge Engineering*, 60(1), pp. 208–221.

Bujel, K., Lai, F., Szczecinski, M., So, W., Fernandez, M., 2018: Solving high volume capacitated vehicle routing problem with time windows using recursive-DBSCAN clustering algorithm. *arXiv preprint arXiv:1812.02300*.

Chao, I. M., Golden, B. L., Wasil, E. A., 1996: A fast and effective heuristic for the orienteering problem. *European Journal of Operational Research*, 88(3), pp. 475–489.

Chen, J., Li, Y., Zhao, L., 2017: Spectral clustering for vehicle routing optimization. *Expert Systems with Applications*, 88, pp. 12–20.

Choi, C., Hong, S. Y., 2021: MDST-DBSCAN: A density-based clustering method for multidimensional spatiotemporal data. *ISPRS International Journal of Geo-Information*, 10(6), p. 391.

Dantzig, G. B., Ramser, J. H., 1959: The truck dispatching problem. *Management Science*, 6(1), pp. 80–91.

Delling, D., Pajor, T., Werneck, R. F., 2013: Round-based public transit routing. *Transportation Science*, 49(3), pp. 591–604.

Desrosiers, J., Dumas, Y., Solomon, M. M., Soumis, F., 1995: Time constrained routing and scheduling. In: *Handbooks in Operations Research and Management Science*, Vol. 8, Elsevier, pp. 35–139.

Dixit, A., Mishra, A., Shukla, A., 2018: Vehicle routing problem with time windows using meta-heuristic algorithms: a survey. In: *Harmony Search and Nature Inspired Optimization Algorithms: Theory and Applications*, ICHSA 2018, Springer, Singapore, pp. 539–546.

Dornemann, J., 2023: Solving the capacitated vehicle routing problem with time windows via graph convolutional network assisted tree search and quantum-inspired computing. *Frontiers in Applied Mathematics and Statistics*, 9, p. 1155356.

El-Sherbeny, N. A., 2010: Vehicle routing with time windows: An overview of exact, heuristic and metaheuristic methods. *Journal of King Saud University – Science*, 22(3), pp. 123–131.

Ester, M., Kriegel, H. P., Sander, J., Xu, X., 1996: A density-based algorithm for discovering clusters in large spatial databases with noise. In: *KDD*, Vol. 96(34), pp. 226–231.

Fritz, P., Berger, J., Brucker, P., 2002: A self-organizing map approach for routing problems with time windows. *International Journal of Neural Systems*, 12(5), pp. 359–370.

Geetha, S., Poonthair, G., Vanathi, P. T., 2009: Improved k-means algorithm for capacitated clustering problem. *INFOCOMP Journal of Computer Science*, 8(4), pp. 52–59.

Gocken, T., Yaktubay, M., 2019: Comparison of different clustering algorithms via genetic algorithm for VRPTW. *International Journal of Simulation Modelling*, 18(4), pp. 574–585.

Goel, R., Maini, R., 2018: A hybrid of ant colony and firefly algorithms (HAFA) for solving vehicle routing problems. *Journal of Computational Science*, 25, pp. 28–37.

Ha, J. M., Moon, G., 2015: An application of k-means clustering to vehicle routing problems. *Journal of Korean Society of Industrial and Systems Engineering*, 38(3), pp. 1–7.

Kuo, Y., Wang, C. C., Hsieh, C. Y., 2008: A hybrid genetic algorithm for the vehicle routing problem with time windows. *Computers & Industrial Engineering*, 54(4), pp. 1117–1132.

Lim, A., Zhang, X., 2007: A two-stage heuristic with ejection pools and generalized ejection chains for the vehicle routing problem with time windows. *INFORMS Journal on Computing*, 19(3), pp. 443–457.

Lin, Y. K., Chen, C. F., Chou, T. Y., 2024: Applying cluster analysis to the vehicle routing problem. In: *International Conference on Flexible Automation and Intelligent Manufacturing*, Springer, Cham, pp. 358–365.

Liu, S., Zhang, J., Wang, W., 2015: Hierarchical clustering algorithm for vehicle routing optimization. *Procedia Computer Science*, 60, pp. 1079–1088.

Lowerre, B. T., 1976: *The harpy speech recognition system*. Doctoral dissertation, Carnegie Mellon University.

Nazari, M., Oroojlooy, A., Snyder, L., Takác, M., 2018: Reinforcement learning for solving the vehicle routing problem. In: *Advances in Neural Information Processing Systems*, 31

Mostafa, N., Eltawil, A., 2017: Solving the heterogeneous capacitated vehicle routing problem using k-means clustering and valid inequalities. In: *Proceedings of the International Conference on Industrial Engineering and Operations Management*, pp. 2239–2249.

Prins, C., 2004: A simple and effective evolutionary algorithm for the vehicle routing problem. *Computers & Operations Research*, 31(12), pp. 1985–2002.

Revanna, J. K. C., Al-Nakash, N. Y. B., 2022: Vehicle routing problem with time window constrain using k-means clustering to obtain the closest customer. *Global Journal of Computer Science and Technology*, 22(D1), pp. 25–37.

Tang, J., Guan, J., Yu, Y., Chen, J., 2014: Beam search combined with MAX-MIN ant systems and benchmarking data tests for weighted vehicle routing problem. *IEEE Transactions on Automation Science and Engineering*, 11(4), pp. 1097–1109.

Villalba, A. F. L., La Rotta, E. C.G., 2020: Comparison of DBSCAN and k-means clustering methods in the selection of representative clients for a vehicle routing model. In: *2020 Congreso Internacional de Innovación y Tendencias en Ingeniería (CONITI)*, IEEE, pp. 1–6.

Zhang, Z., Tan, S., Qin, J., Zou, K., Zhou, S., 2025: Multi-strategy ant colony optimization with k-means clustering algorithm for capacitated vehicle routing problem. *Cluster Computing*, 28(3), pp. 1–24.

Žunić, E., Delalić, S., Đonko, D., 2022: Adaptive multi-phase approach for solving the realistic vehicle routing problems in logistics with innovative comparison method for evaluation based on real GPS data. *Transportation Letters*, 14(2), pp. 143–156.