

Improved reflectance calculation in full-waveform LiDAR considering the angle of incidence

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Abstract

Reflectance is a widely used feature for all types laser scanning data. Thus, the accuracy and improvement of the reflectance parameter is a persistent topic of research. For short laser pulses with medium-sized footprints, previous work has investigated the effects of inclined targets on the recorded waveform of full-waveform LiDAR systems. In this work, a new methods to extract incidence angle from only a single waveform can be leveraged to improve reflectance values through recalculation based on the laser-radar equation and correcting for angle of incidence artifacts. The results of the proposed method are evaluated with two datasets based on two different topo-bathymetric laser scanners. For both systems, we calculated the relative biconical reflectance and relative averaged bidirectional reflectance distribution function (rBRDF) and evaluated them on homogeneous roof faces. The two reflectance measures are then compared to the initial reflectance values of the laser scanners used in the study. Both measures showed improvements compared to the standard values. The biconical reflectance shows the best overall mean score for all surveyed roofs with an median absolute deviation improvement of 0.80 dB to 0.62 dB for the *RIEGL VQ-880-GII* and 0.61 dB to 0.56 dB for the *RIEGL VQ-1560i-DW*. In addition the rBRDF also displays an improvement with varying results depending on the deployed system. These results highlight the advantages of the proposed reflectance measures and the potential improvement of the widely used LiDAR attribute.

1. Introduction

Airborne laser scanning has seen constant improvement of sensor technology and data processing over the last decade, as the use of light detection and ranging (LiDAR) data has become a common tool among all fields of surveying (Mandlbürger et al., 2023; Ullrich et al., 2007). For the wide range of available systems, improvements in both sensor and platform technology have allowed the survey of vast natural landscapes and urban areas (Pfennigbauer et al., 2014; Ullrich et al., 2007). In addition to the long-standing use of airplanes and helicopters, the emergence of unmanned aerial vehicles (UAVs) as carrier platforms has further reduced entrance barriers for airborne LiDAR-based surveys (Mandlbürger et al., 2011; Pfennigbauer and Ullrich, 2011).

Depending on the area surveyed, there are different wavelengths used by the laser scanning system. For topographic laser scanning, most sensors use near-infrared (NIR) light, while for bathymetric applications typically green lasers are used (Mallet and Bretar, 2009; Pfennigbauer and Ullrich, 2011). Furthermore, bathymetric LiDAR uses larger laser footprint sizes due to general regulations for eye safety and shorter pulse durations (1.5 ns) for better separation of water surface and shallow water bottom (Mandlbürger et al., 2023, 2011; Williams Jr, 2017). In contrast, the typical NIR LiDAR has a small footprint at ground level and longer pulses (3 ns).

For both topographic and bathymetric systems, the introduction of full-waveform laser scanning has enabled additional signal processing that goes beyond peak detection (Mallet and Bretar, 2009; Pfennigbauer et al., 2014; Rhomberg-Kauert et al., 2024). There, the full-waveform refers to the recoding of the entire sig-

nal amplitude history of the laser pulse echo over time (Mallet and Bretar, 2009; Ullrich et al., 2007; Ullrich and Pfennigbauer, 2011). The recorded waveform can be characterized by the scattering characteristics and the geometry of the target (Jutzi and Gross, 2009; Kaasalainen et al., 2011). On the geometric side, the range to the reflecting object can be derived by estimating the time it takes for laser pulses to travel to a target and back (Guo et al., 2023; Wagner, 2010). The characteristic of scattering can be described by radiometry, where multiple attributes can be derived from the full-waveform analysis (Jutzi and Gross, 2009; Pfennigbauer et al., 2013). Two of the most prominent attributes are the amplitude and reflectance (Haner and Menzies, 1989; Pfennigbauer and Ullrich, 2010). The amplitude is the signal strength measured by the receiver and the reflectance is a property of the reflecting object, in general. These two attributes are also related to the width of the echo pulse (Wagner et al., 2006), since the reflected energy of the target is proportional to the maximum amplitude and the echo pulse at full width half maximum (FWHM) (Wagner et al., 2006).

Both the amplitude and the width of the echo pulse are affected by the material of the target and the angle of incidence of the laser signal (Bolkas, 2019; Kukko et al., 2008; Roca-Pardiñas et al., 2014). For Lambertian scatters, the intensity of the return signal decreases with cosine of the angle, where a zero incidence angle corresponds to a equivalence between the normal and laser beam vector. (Abed et al., 2012; Kukko et al., 2008; Roca-Pardiñas et al., 2014). To compensate for this, different methods have been proposed (Kaasalainen et al., 2011; Kashani et al., 2015; Bolkas, 2019). Most of these focus on laboratory experiments, where environmental factors can be controlled. Furthermore, the methods proposed in the literature vary greatly in terms of complexity and range from single-parameter func-

tions to complex models (Carrea et al., 2016; Kaasalainen et al., 2011; Kashani et al., 2015; Shaker et al., 2012).

Another parameter that describes the radiometric properties is the reflectance. It is a relative measure based on a-priori or a-posteriori calibration. Airborne and terrestrial laser scanners commonly utilize laboratory measurements to transform the in-situ measured returned energy into the calculated reflectance for each measured point (Pfennigbauer and Ullrich, 2010, 2011; Briese et al., 2012; Mandlbürger, 2024). The reflectance parameter is given in dB with respect to a reference target, e.g. a white diffusely reflecting target, typically represented with 0 dB (Pfennigbauer and Ullrich, 2010, RIEGL, 2009, 2012). The reflectance of laser return is then expressed proportionally to the reference target, with a value of -3 dB, for example, corresponding to 50% of diffuse reflectance, and a value greater than zero implying a target that is reflecting more light than a diffuse target, for example, a partially retro-reflecting surface (Pfennigbauer and Ullrich, 2010, RIEGL, 2009).

In the work of Rhomberg-Kauert et al. (2024), the topic of changes in the shape of the waveform was analyzed after interaction with angled targets and a method was introduced to estimate the incidence angle that does not rely on the neighborhood, but purely on the waveform analysis. While established neighborhood-based methods first derive the target's normal direction and calculate the incidence angle in relation to the laser beam direction, the method introduced in Rhomberg-Kauert et al. (2024) directly calculates the incidence angle from the returned full waveform. The objective of this study is to first introduce a new approach to calculate the reflectance values de-

rived from the radar equation (Wagner et al., 2006) and then to compare different methods to estimate the angle of incidence as input for the new reflectance measures. We first present the radar equation in detail (Section 2.3) and introduce two possible reflectance measures (Section 3.2) based on (Wagner et al., 2006). Then we calculate these reflectance measures for both the waveform- and the neighborhood-based angle of incidence estimation and present the results in Section 4. The results are compared using standard metrics for a selection of differently angled roofs. This is then critically discussed and connected to the literature (Section 5).

2. Materials

For the evaluation of the corrected reflectance values, we first introduce the evaluation datasets and outline the mathematical background. We used two different airborne laser scanning datasets acquired with a bathymetric (green) scanner. In addition a terrestrial laser scanning (TLS) dataset for a small subset of the area is used to provide additional validation.

2.1 Study site

The surveyed area is in Loosdorf (48.2010° N, 15.4004° E; WGS 84) in Lower Austria. The region contains a variety of structures with different angles, such as houses with tilted roofs (Figure 1), which can later be used to evaluate reflectance values for differently inclined surfaces. The acquired data are split into two reports. The main study area (Figure 1 B), which contains the multiple buildings and a smaller area of interest for a secondary validation using TLS data.



Figure 1. (A) Map of the village of Loosdorf in Lower Austria (Bundesamt für Eich- und Vermessungswesen, 2024) and (B) orthophoto map of the main study site. (C) Selected region where additional TLS data was acquired for reflectance comparison.

2.2 LiDAR data

The two main datasets of this study were acquired with the *RIEGL VQ-880-GII* topo-bathymetric laser scanner (Sensor I) and the *RIEGL VQ-1560i-DW* dual-wavelength laser scanner (Sensor II). For each system, the parameters are shown in Table 1. Both systems use a wavelength of 532 nm. Hence, the main difference between both systems is that Sensor I features a constant off-nadir angle of 20° (RIEGL, Laser Measurement Systems GmbH, 2022b), resulting in a circular scan pattern on the ground, while Sensor II delivers a pair of straight parallel scan lines approximately orthogonal to the flight direction with an angular offset of ±14° (RIEGL, Laser Measurement Systems GmbH, 2022a). The off-nadir angles vary between ±30° (Figure 2).

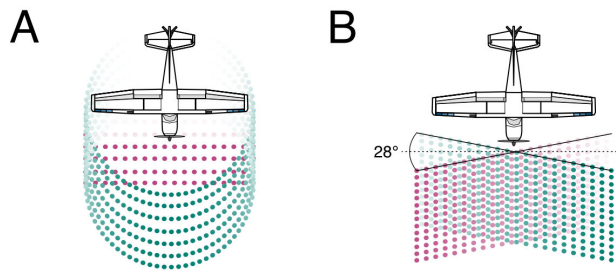


Figure 2. (A) Illustration of the circular scan (green channel) and linear (NIR channel) pattern of the Sensor I, where the green channel has a nadir angle of 20°, (B) The two linear scan patterns (green and NIR channel) of the Sensor II.

The scan parameters, such as the wavelength λ , beam divergence θ and footprint diameter at ground level $2\mathcal{F}_0$, and pulse width at full width half maximum (FWHM) are displayed in Table 1. Both scanners support full-waveform recording, allowing recording of the full temporal signal amplitude record for each point. In this study, point extraction from the full laser waveform is performed in post-processing using Gaussian decomposition (Ullrich and Pfennigbauer, 2011), as this provides also the echo pulse width (FWHM) which serves as input for the angle of incidence estimation for System I and II.

System	λ [nm]	θ [mrad]	$2\mathcal{F}_0$ [m]	τ_r [ns]
Sensor I	532	0.9	0.4-0.7	1.5
Sensor II	532	1.0	0.6-1.0	1.5
TLS	1064	0.25 0.35*	0.005 – 0.05**	3.0

Table 1. The properties of the three LiDAR systems employed in the study. (*) Beam divergence according to the datasheet. (**)

Footprint estimated from scanner tripod height and rounding effect in the point cloud .

In addition, for the study site shown in Figure 1 C, an additional TLS dataset was acquired using the *RIEGL VZ-600i* laser scanner. The area was surveyed from multiple scan positions, thus providing a high resolution laser scan of the site with a significantly smaller footprint compared to the airborne laser scanning data (Table 1).

2.3 Mathematical background

To recalculate reflectance considering the angle of incidence, we first introduce the mathematical terminology and present the equations to calculate the various radiometric quantities. To ensure consistent terminology, we introduce the variables mentioned throughout the article in Table 2.

Symbol	Property	Terminology
c		Speed of light [m/s]
t		Time in seconds [s]
R	$R = \frac{1}{2}c \cdot t$	Range [m]
θ		Beam divergence [mrad]
α		Angle of incidence [°]
r_0		Footprint radius at $R=0$ [m]
$\mathcal{F}_0$	$\mathcal{F}_0 = \sqrt{\left(\frac{\theta \cdot ct}{2}\right)^2 + r_0^2}$	Footprint radius at ground [m]
P_r	Equation 1	Power recorded [dB]
P_e		Power emitted [dB]
Ω_{BSS}		Angle of back-scattered energy [rad]
A	$A = \pi * \mathcal{F}_0^2$	Area of the scattering object [m ²]
D_r		Diameter of the laser receiver [m]
σ		Backscatter cross-section [m ² sr ⁻¹]
$\hat{A}$		Amplitude of the system waveform [W]
τ_e		Pulse width of the system waveform [s]
$\hat{P}$		Amplitude of the recorded laser pulse [W]
τ_r		Pulse width of the recorded laser pulse [s]
η_{sys} η_{atm}		System loss Atmosphere loss
ρ		Reflectance [dB]
$\mathcal{P}$	$p \in \mathcal{P}$	The set of all LiDAR points

Table 2. Table explaining the variables used in the article, their discription and units.

The interaction of the laser pulse with an inclined surface can be described by the laser-radar equation (Equation 1), which accounts for the reflection, absorption, and scattering of the measured object. Let P_e be the power emitted by the laser, R the distance from the origin of the laser to the target, A the area of the scattering object, D the diameter (aperture) of the laser receiver, ρ the reflectivity of the target, and Ω_{BSS} the angle into which the backscattered energy of the target is reflected (backscattering solid angle), then the received power P_r can be calculated (Guo et al., 2023; Wagner et al., 2006). The calculation of the received power can also be adjusted from a vacuum setting to real-world recordings by multiplying the loss term of the system η_{sys} and the atmospheric loss term η_{atm} , which produces the final equation (Wagner et al., 2006; Wagner, 2010). It should be noted that the radar equation does not account for the material properties of the target surface. Thus, e.g. roughness of the target causes an uncertainty in the calculated received power.

$$P_r = \frac{4P_e}{\pi\theta^2 \cdot R^2} \cdot \frac{A \cdot \rho}{\Omega_{BSS} \cdot R^2} \cdot \frac{\pi D_r^2}{4} \cdot \eta_{sys} \cdot \eta_{atm} \quad (1)$$

$$= \frac{P_e}{\pi\theta^2 \cdot R^2} \cdot \frac{1}{R^2} \cdot \frac{D_r^2}{4} \cdot \frac{4\pi \cdot \rho \cdot A}{\Omega_{BSS}} \cdot \eta_{sys} \cdot \eta_{atm} \quad (2)$$

$$= \frac{P_e \cdot D_r^2}{4\pi\theta^2 \cdot R^4} \cdot \underbrace{\frac{4\pi}{\Omega_{BSS}} \rho A}_{=\sigma} \cdot \eta_{sys} \cdot \eta_{atm} \quad (3)$$

$$= \frac{P_e \cdot D_r^2}{4\pi\theta^2 \cdot R^4} \cdot \sigma \cdot \eta_{sys} \cdot \eta_{atm} \quad (4)$$

$$= P_e \cdot \frac{D_r^2}{4\pi R^4 \cdot \theta^2} \cdot \sigma \cdot \eta_{sys} \cdot \eta_{atm} \quad (5)$$

For the simplified radar equation shown in Equation 5, the received power P_r can be calculated through the emitted power P_e , diameter of the laser receiver D_r , beam divergence θ , range R , backscatter cross-section of the target σ and the loss parameters η_{sys} and η_{atm} .

3. Methods

From the laser-radar equation two different reflectance measures can be deduced. First, the biconical reflectance and, second, the averaged bidirectional reflectance distribution function (BRDF). The mathematical equations for these two measures are outlined in the following section, together with the metrics used in Section 4, outlining the proposed approach for the calculation of angle of incidence corrected reflectance values.

3.1 Angle of incidence calculation

To correct artifacts related to the angle of incidence, we introduce the method used to calculate these angles. In this study, we focus on two approaches, first neighborhood-based and second waveform-based angle of incidence estimation.

For the neighborhood-based method, we use the point cloud processing software OPALS (Pfeifer et al., 2014). There, the module opalsNormals is used with k-nearest neighbor search (k=32) and quadrant-wise point selection mode, which results in 8 points in each quadrant around a LiDAR data point to estimate the surface normal vectors. We estimate the normal vectors using a robust plane fitting approach, which detects and eliminates outliers within a robust least-squares adjustment framework. The angle of incidence are then calculated as the scalar product of these normal vectors and the laser beam vectors, derived from the flight trajectory.

The waveform-based angle of incidence estimation is built on the simulation framework of Rhomberg-Kauert et al. (2026), which establishes a relation between the width of the echo pulse and the angle of incidence. We apply the simulation of Rhomberg-Kauert et al. (2026) in accordance with the parameters of Sensor I and Sensor II and obtain a lookup table of the echo pulse width and the corresponding angle of incidence for each sensor. Thus, for each point the FWHM of the echo waveform extracted during post-processing (Section 2.2) can be used to estimate an angle of incidence based solely on the recorded waveform. To exclude potential influences from multiple echoes, only single-echo points are analyzed.

3.2 Reflectance calculation

To calculate reflectance values, this study expands on the work of Wagner (2010) and Wagner et al. (2006) on radiometric calibration. For a Gaussian pulse and a Lambertian reflector, Equation 5 can be transformed into

$$\hat{P} = \frac{\hat{A} \cdot \tau_e}{\tau_r} \cdot \frac{D_r^2}{4\pi R^4 \cdot \theta^2} \cdot \sigma \cdot \eta_{sys} \eta_{atm}, \quad (6)$$

where σ is the laser-radar cross-section, $\hat{P}$ and τ_r are the amplitude [W] and width [s] of the recorded laser echo, and $\hat{A}$ and τ_e the amplitude [W] and width [s] of the system waveform. The only difference between the emitted power P_e of Equation 5 and Equation 6 is the substitution through $\hat{A} \cdot \tau_e$ in addition to the substitution of the received power P_r through $\hat{P} \cdot \tau_r$, which corresponds to the transformation of the pulse energy to the area underneath the curve (of a Gaussian pulse). These parameters form the basis for the calculation of the biconical reflectance and relative averaged BRDF (rBRDF). By first defining the backscattering coefficient [m^2/m^2] γ according to Wagner (2010) with the exception of a different variable for the beam width θ (Equation 18 in Wagner (2010)), we can calculate the two reflectance measures.

$$\gamma = \frac{4\sigma}{\pi R^2 \theta^2}, \quad (7)$$

where R is the range and θ the beam divergence. Substituting the cross-section σ for the backscattering coefficient γ in Equation 6 yields

$$\gamma = \frac{16R^2 \hat{P} \cdot \tau_r}{\hat{A} \tau_e \cdot D_r^2 \cdot \eta_{sys} \eta_{atm}} \quad (8)$$

$$= \frac{16}{D_r^2 \hat{A} \tau_e \cdot \eta_{sys} \eta_{atm}} R^2 \hat{P} \cdot \tau_r \quad (9)$$

$$= \underbrace{\frac{16}{D_r^2 \tau_e \cdot \eta_{sys}}}_{C_{cal}} \frac{R^2 \hat{P} \cdot \tau_r}{\hat{A} \cdot \eta_{atm}}. \quad (10)$$

This can be summarized into a calibration coefficient and the variables measured by the LiDAR system (Wagner, 2010).

$$\gamma = C_{cal} \frac{R^2 \hat{P} \cdot \tau_r}{\hat{A} \cdot \eta_{atm}} \quad (11)$$

For Lambertian targets, the backscattering coefficient γ can be related to the biconical reflectance ρ and BRDF $\langle f \rangle$ (Wagner, 2010).

$$\gamma = \frac{4\pi}{\Omega_{BSS}} \underbrace{\rho(\Omega_i, \Omega_{BSS})}_{\tilde{\rho}} = \frac{4\pi}{\Omega_{BSS}} \tilde{\rho} = 4\pi \langle f \rangle \cos(\alpha), \quad (12)$$

where Ω_i and Ω_{BSS} are the solid angles of the transmitter and target-reflection cones and α is the angle of incidence in alignment with Wagner (2010). To improve the notation and variable management, we introduce here the variable $\tilde{\rho}$ for the biconical reflectance $\rho(\Omega_i, \Omega_{BSS})$ to simplify Equation 12. Substituting

the backscattering coefficient for the two reflectance measures, we can convert Equation 11 to

$$\begin{aligned}\tilde{\rho} &= C_{\text{cal}} \frac{R^2 \cdot \hat{P} \cdot \tau_r}{\hat{A} \cdot \eta_{\text{atm}}} \cdot \frac{\Omega_{\text{BSS}}}{4\pi} \\ &= C_{\text{cal}} \frac{R^2 \cdot \hat{P} \cdot \tau_r}{\hat{A} \cdot \eta_{\text{atm}}} \cdot \frac{\pi D^2}{16\pi R^2}\end{aligned}\quad (13)$$

$$\langle f \rangle = C_{\text{cal}} \frac{R^2 \cdot \hat{P} \cdot \tau_r}{\hat{A} \cdot \eta_{\text{atm}}} \cdot \frac{1}{4\pi \cos(\alpha)}.\quad (14)$$

To obtain relative reflectance measures, which relate to the calibration of the LiDAR systems during initial setup, we normalize both the biconical reflectance $\tilde{\rho}$ and the BRDF $\langle f \rangle$. To obtain the value of the calibration constant R_{cal} , the recorded range R is a match using a lookup-table. Due to the matching of values, R and R_{cal} are equal in the calculation. Furthermore, assuming that the initial amplitude $\hat{A}$ and receiver aperture D are the same for both calibration and measurement, we finally obtain the following relative measures:

$$\begin{aligned}\frac{\tilde{\rho}}{\tilde{\rho}_{\text{cal}}} &= \frac{C_{\text{cal}} \frac{R^2 \cdot \hat{P} \cdot \tau_r}{\hat{A} \cdot \eta_{\text{atm}}} \cdot \frac{\pi D^2}{16\pi R^2}}{C_{\text{cal}} \frac{R_{\text{cal}}^2 \cdot \hat{P}_{\text{cal}} \cdot \tau_r^{\text{cal}}}{\hat{A} \cdot \eta_{\text{atm}}} \cdot \frac{\pi D_{\text{cal}}^2}{16\pi R_{\text{cal}}^2}} \\ &= \frac{\hat{P} \cdot \tau_r}{\eta_{\text{atm}}} \cdot \left(\frac{\hat{P}_{\text{cal}} \cdot \tau_r^{\text{cal}}}{\eta_{\text{atm}}^{\text{cal}}} \right)^{-1}\end{aligned}\quad (15)$$

$$\begin{aligned}\frac{\langle f \rangle}{\langle f \rangle_{\text{cal}}} &= \frac{C_{\text{cal}} \frac{R^2 \cdot \hat{P} \cdot \tau_r}{\hat{A} \cdot \eta_{\text{atm}}} \cdot \frac{1}{4\pi \cos(\alpha)}}{C_{\text{cal}} \frac{R_{\text{cal}}^2 \cdot \hat{P}_{\text{cal}} \cdot \tau_r^{\text{cal}}}{\hat{A} \cdot \eta_{\text{atm}}^{\text{cal}}} \cdot \frac{1}{4\pi \cos(0)}} \\ &= \frac{R^2 \cdot \hat{P} \cdot \tau_r}{\eta_{\text{atm}} \cos(\alpha)} \cdot \left(\frac{R_{\text{cal}}^2 \cdot \hat{P}_{\text{cal}} \cdot \tau_r^{\text{cal}}}{\eta_{\text{atm}}^{\text{cal}}} \right)^{-1}\end{aligned}\quad (16)$$

Here, $\tilde{\rho}/\tilde{\rho}_{\text{cal}}$ and $\langle f \rangle/\langle f \rangle_{\text{cal}}$ are the relative measures calculated on the basis of the data measured by the LiDAR system, with the calibration parameters labeled $[\]_{\text{cal}}$. To convert these relative measures to values given in dB, a $10 \log_{10}$ transformation is applied to both sides of the equations. For the relative biconical reflectance, this yields

$$\tilde{\rho} = \frac{\hat{P} \cdot \tau_r}{\eta_{\text{atm}}}, \quad \tilde{\rho}_{\text{cal}} = \frac{\hat{P}_{\text{cal}} \cdot \tau_r^{\text{cal}}}{\eta_{\text{atm}}^{\text{cal}}}\quad (17)$$

$$\Rightarrow \rho_{\text{rel}} = 10 \cdot \log_{10} \left(\frac{\tilde{\rho}}{\tilde{\rho}_{\text{cal}}} \right)\quad (18)$$

and for the rBRDF,

$$\langle f \rangle = \frac{R^2 \cdot \hat{P} \cdot \tau_r}{\eta_{\text{atm}} \cos(\alpha)}, \quad \langle f \rangle_{\text{cal}} = \frac{R_{\text{cal}}^2 \cdot \hat{P}_{\text{cal}} \cdot \tau_r^{\text{cal}}}{\eta_{\text{atm}}^{\text{cal}}}\quad (19)$$

$$\Rightarrow \langle f \rangle_{\text{rel}} = 10 \cdot \log_{10} \left(\frac{\langle f \rangle}{\langle f \rangle_{\text{cal}}} \right).\quad (20)$$

This calculation is now solely based on calibration and measured data, providing a new reflectance measure for the system.

3.3 Waveform averaging

To later expand the analysis of the reflectance correction in relation to the recorded signal, this section presents waveform averaging as a method to outline general trends in the LiDAR signal from multiple combined recordings. The methods used is

based on the study of (Rhombert-Kauert et al., 2025), where the recorded waveforms can be averaged to outline general trends across a target. In the case of this study, similar to the usage in (Rhombert-Kauert et al., 2026), the waveforms reflected from the same target are aligned by the first recorded sample with amplitude greater than 30 ADC. This provides multiple amplitudes, one from each waveform, approximately every 0.5 ns, which can then be combined using the average across the amplitudes for each point in time. The discrete representation of the averaged waveform of the target can then be interpolated to a continuous curve.

3.4 Evaluation metric

For the evaluation of both the initial and newly calculated reflectance values, we introduce three measures often associated with distributional dispersion: (i) the variance coefficient (CV), (ii) the median absolute deviation (MAD), and (iii) the interquartile range (IQR). Let $\varrho(\mathcal{P})$ be the distribution of reflectance values with mean $\mu_{\mathcal{P}}$, median $\bar{\mu}_{\mathcal{P}}$, standard deviation $\sigma_{\mathcal{P}}$, first quartile $Q_{\mathcal{P},1}$, and third quartile $Q_{\mathcal{P},3}$. The evaluation measures are then defined as

$$\text{CV}(\varrho(\mathcal{P})) = \frac{\sigma_{\mathcal{P}}}{\mu_{\mathcal{P}}},\quad (21)$$

$$\text{MAD}(\varrho(\mathcal{P})) = \text{Median}_{p \in \mathcal{P}} (|\varrho(p) - \bar{\mu}_{\mathcal{P}}|),\quad (22)$$

$$\text{IQR}(\varrho(\mathcal{P})) = Q_{\mathcal{P},3} - Q_{\mathcal{P},1}.\quad (23)$$

3.5 Evaluation of roof sections

For the evaluation of all roofs in the surveyed area, the data have to be split into individual roof sections. The study area offers a variety of different roofs, some spatially separated and others within close proximity. Therefore, we use density clustering to group the roofs of individual buildings. In particular, the roofs are grouped using DBSCAN (Buitinck et al., 2013) with an epsilon parameter of 2 and 1.5 for Sensor I and Sensor II, respectively.

4. Results

With the introduced LiDAR datasets, we can now evaluate the improvements made by the new reflectance measures. For this, we evaluate both the waveform-based and the neighborhood-based angle of incidence, for each metric, and compare the new values to the manufacturer's reflectance output.

The section is split into three parts: (i) First, the differences in recalculated reflectance measures are shown for selected groups of roofs, highlighting the differences between the LiDAR systems and providing insight into the differences of reflectance measures (Section 4.1). (ii) Second, a general evaluation is given for all roofs in the area of interest, for both datasets (Section 4.2). (iii) Third, the comparison of the TLS and airborne laser scanning data (Section 4.3).

4.1 Evaluation for selected roofs

The results and limitations of the different reflectance measures both become visible in the visual comparison when using all points on the roofs including points from multi-return laser shots (Figure 3) and, in contrast, the application of the method to single-return echo pulses with a minimum FWHM of 1.5 ns in Figure 4. This differentiation is done to provide an improved

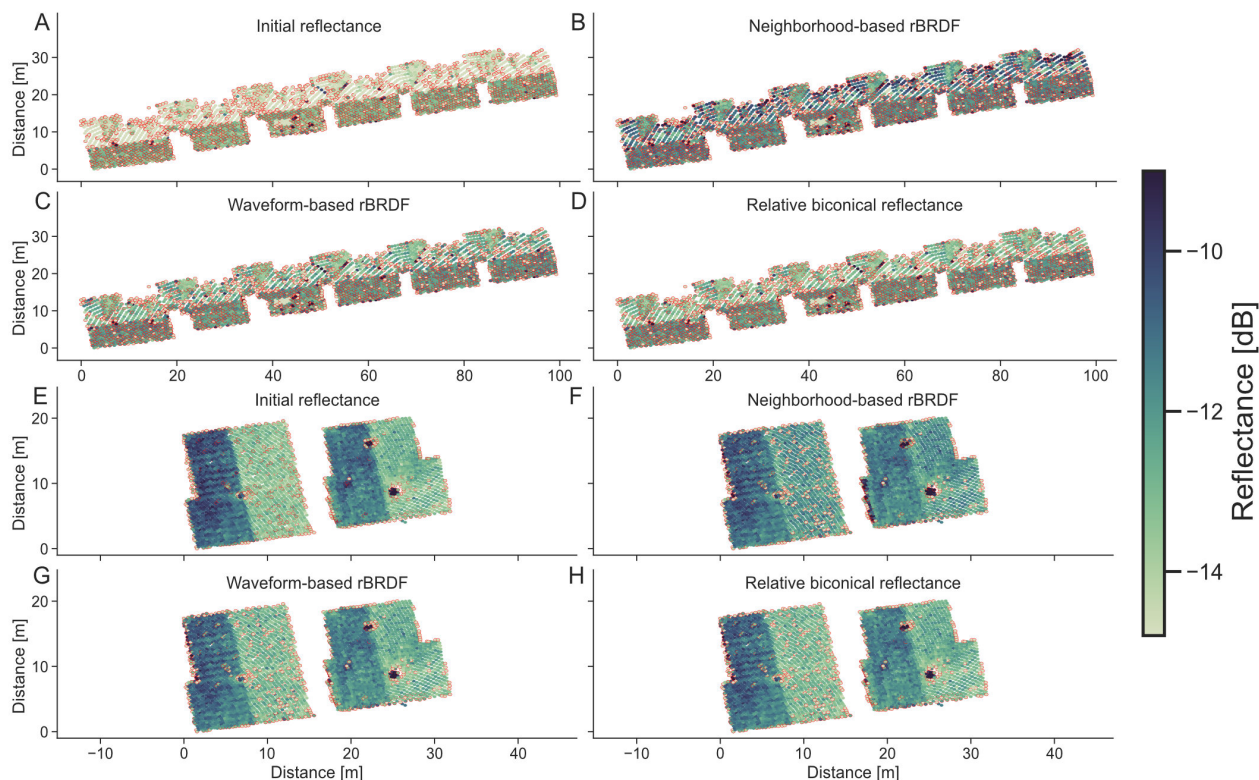


Figure 3. Manufacturer's reflectance values (A and E), neighborhood-based rBRDF (B and F), waveform-based rBRDF (C and G) and relative biconical reflectance (D and H) for selected roofs of Sensor I (A-D) and Sensor II (E-H). For all panels, the red edge color indicates points with a FWHM lower than 1.5 ns (could not be extracted during processing) or laser shots with more than one echo.

visualization, as both for System I and System II not every point on the roof meets this criterion and thus the visualization of sparse point cloud can be avoided (Figure 3).

While for Sensor I the relative biconical reflectance displays the best results and shows a high similarity to the waveform-based rBRDF, the neighborhood-based rBRDF also provides a substantial improvement compared to the initial reflectance values. In contrast, for Sensor II the improvements through recalculation of the reflectance values are less distinct. There, the neighborhood-based rBRDF displays the most homogeneous reflectance across the roofs, while the waveform-based rBRDF and relative biconical reflectance display a high degree of outliers due to the Gaussian fitting (where the fitting did not converge) for the Sensor II during the processing.

In addition, the difference in the scan pattern and thus the angles of incidence can also be seen in Figure 3. For Sensor I, the majority of indices angles is between 22° to 54° and for Sensor II between 14° to 38° . Therefore, the variation of reflectance values is less for Sensor II, as the linear scan pattern leads to lower angles of incidence in the study area. The study area is only compromised of the center section of the flight strip and thus the varying angle of Sensor II does not change as much, compared to the entire width of the flight strip. In contrast, the Palmer scan pattern with an off-nadir angle of 20° leads to higher angles of incidence for the side of the roofs facing the same way as the flight direction (where roof edge is orthogonal to the flight direction).

Similar results can also be seen in the distribution of the reflectance values (Figure 4), where the red encircled points were not taken into account. For the manufacturer's reflectance values,

one can see a broad variation of reflectance values for Sensor I, while the recalculated reflectance values show a narrower distribution. The waveform-based rBRDF and relative biconical reflectance display the smallest standard deviations, while the neighborhood-based rBRDF displays a wider distribution. The smaller reflectance deviations across a single roof would be expected, as the roof consists mainly of the same material and thus should have the same reflectance properties for each house. In contrast, for Sensor II, the neighborhood-based rBRDF displays a distribution with the smallest standard deviation. In comparison, the manufacturer's reflectance values of Sensor II display a split distribution, which is related to the varying angles of incidence of the roofs (Figure 3). Furthermore, the waveform-based rBRDF and relative biconical reflectance display a distribution similar to the manufacturers reflectance values, with slightly more homogeneous distributions. Thus, the newly calculated reflectance measures indicate an improvement for both systems, where differences are most likely due to the scan pattern and the system settings. These differences in the angles of incidence in combination with the limitations during the Gaussian fitting for Sensor II have to be considered during the general evaluation.

4.2 General evaluation

The quantitative evaluation of all clustered roofs in the study area is presented in Figure 5. For each row, the median reflectance metric is represented by the black line; thus, a box plot with a median line positioned below the black line indicates an improvement for the calculated reflectance values. Such improvements can clearly be seen for the MAD and IQR of the waveform-based rBRDF and relative biconical reflect-

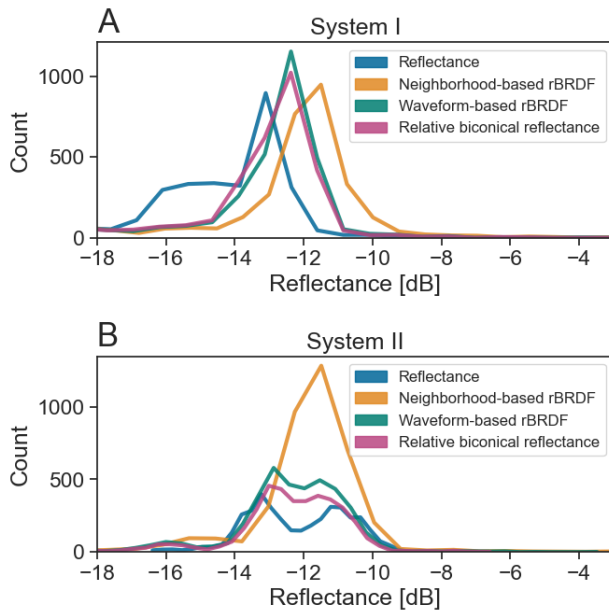


Figure 4. Manufacture's reflectance, neighborhood-based rBRDF, waveform-based rBRDF and relative biconical reflectance distributions for the roofs shown in Figure 3, for Sensor I (A) and Sensor II (B).

ance of Sensor I (Figure 5). For Sensor II these improvements are smaller. MAD and IQR show a similar median improvement, while neighborhood-based rBRDF displays a lower decrease (Figure 5). For CV, the neighborhood-based rBRDF displays worse results than the other two reflectance measures, but, in general, all three measures show worse median reflectance scores (Figure 5).

When comparing the mean scores for each metric (Table 3), the improvements in the recalculated reflectance can be seen for both sensors. The relative biconical reflectance displays the overall best scores across both sensors. However, for the MAD and IQR also the neighborhood-based and waveform-based rBRDF display a significant improvement. This aligns with the results shown in Figure 3. The generally higher incidence angles of Sensor I also lead to a greater improvement in the measures, as with higher angles the effects of the incidence angles on the reflectance become stronger. Therefore, for both angle estimation methods, the improvements are above the mean for both rBRDF and relative biconical reflectance (Table 3).

Sensor	Metric	Neighborhood Waveform Biconical			
		ρ	$\langle f \rangle_{rel}$	$\langle f \rangle_{rel}$	ρ_{rel}
Sensor I	VC	0.10	0.17	0.12	0.11
Sensor II	VC	0.10	0.18	0.12	0.11
Sensor I	MAD	0.80	0.80	0.65	0.62
Sensor II	MAD	0.61	0.56	0.56	0.56
Sensor I	IQR	1.85	1.72	1.37	1.34
Sensor II	IQR	1.38	1.24	1.26	1.25
Mean		0.81	0.78	0.68	0.67

Table 3. Mean values of the evaluation scores shown in Figure 5 and mean across all metrics for each reflectance measure.

4.3 Comparison to TLS data

In addition to the general evaluation of the main area of interest, the TLS dataset can be used in an independent evaluation of

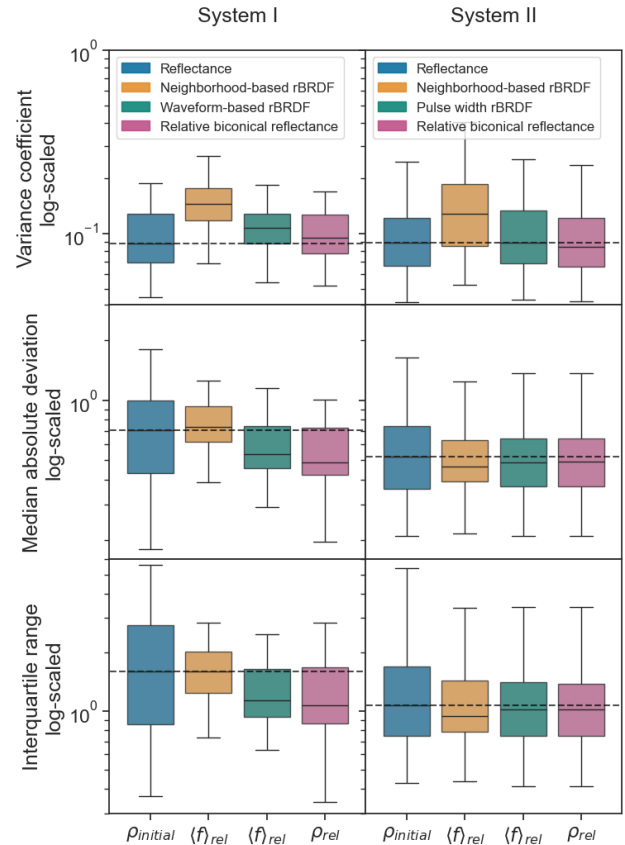


Figure 5. Box plots for the different metrics used to evaluate the reflectance calculation of Sensor I and II. Each row displays one of the three selected metrics: (i) CV, (ii) MAD, and (iii) IQR. The box-plots display the manufacturer's reflectance for both systems, the rBRDF $\langle f \rangle_{rel}$ (for both neighborhood-based and waveform-based angles) and the relative biconical reflectance ρ_{rel} . For all rows, the black line represents the median manufacturer's reflectance.

the reflectance measures. For this, the reflectance values are again computed for all measures, with the limitation to the area surveyed by the TLS. Furthermore, the rBRDF can be computed for the neighborhood-based angles of incidence using the denser TLS point cloud. These high resolution angles of incidence are then mapped to the closest neighbor of the airborne LiDAR point cloud, and from there the computation proceed as before.

To evaluate the reflectance measure in comparison to the smaller footprint TLS dataset, the reflectance values of the TLS point count are also matched to their nearest neighbors in the airborne LiDAR point cloud. Using these values, the absolute difference between the initial airborne and new reflectance measures (rBRDF and biconical) can be computed. The results of this analysis are shown in Figure 6. There, the new reflectance measures clearly outperform the initial reflectance measured for the roofs surveyed. However, for the street, the new reflectance measures perform worse than the manufactures reflectance measure. This is related to the usage of Gaussian decomposition in point cloud extraction, where the street data show a low incidence angle throughout the surveyed area (Figure 6 B and D). The low angle of incidence leads to a relatively undistorted return of the emitted signal, which, besides the main return, also consists of a ringing effect (Figure 7). Therefore,

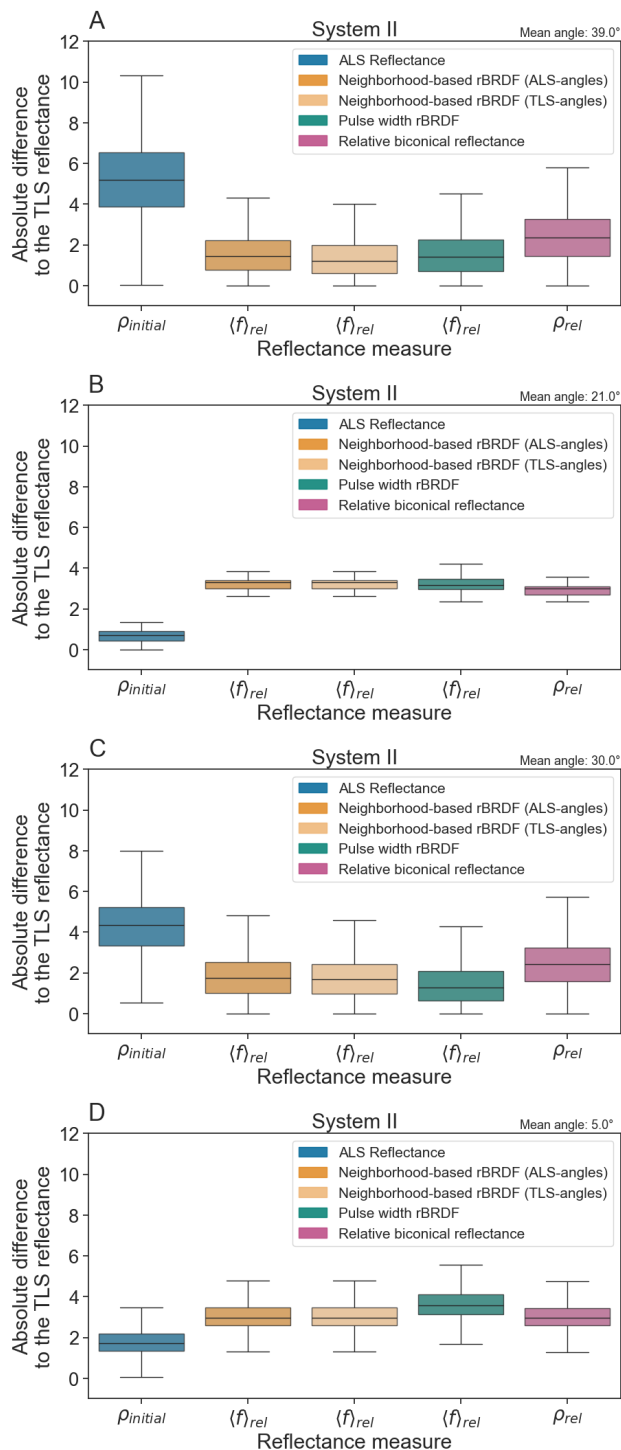


Figure 6. Comparison between the TLS reflectance values, the initial reflectance and presented reflectance measure. The Y-axis displays the absolute difference between the point wise reflectance values of the TLS data and each reflectance measure listed on the X-axis; the Manufacturer's reflectance, neighborhood-based rBRDF (ALS-based angles of incidence), neighborhood-based rBRDF (TLS-based angles of incidence), waveform-based rBRDF and relative biconical reflectance distributions. (A-C) Display the results for the roofs shown in Figure 1 C and (B-C) for the street shown in Figure 1 C, for Sensor I (A) and Sensor II (B).

the Gaussian fitting captures the pulse width and amplitude of the main echo, but not the entire signal echo, and thus the new reflectance measures inadequately correct the reflectance values.

For the roof data, the best results are achieved by the rBRDF, either for the TLS neighborhood-based or pulse width-based angles of incidence. Although both for System I and System II the lower score of the best to the second best measure have marginal differences.

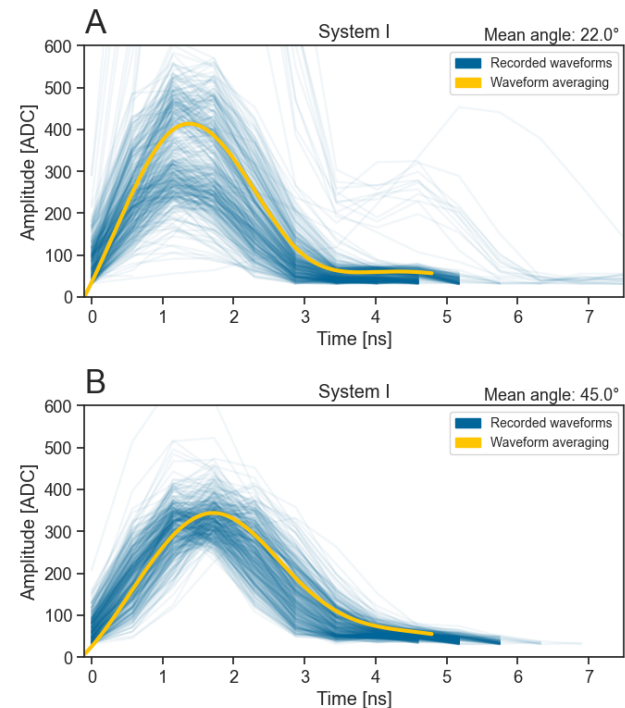


Figure 7. Waveforms and averaged waveform for the roof and street dataset of System I.

5. Discussion

The evaluation presented the comparison of standard reflectance values, generated during echo extraction, with the newly calculated rBRDF and the relative biconical reflectance. These results show that the new reflectance values improve for most metrics, but differ in the extent for the different reflectance measures and metrics. Therefore, this section critically discusses these results in light of recent work.

5.1 Reflectance calculation

The use of real-world data creates a general limit to the evaluation, as differences in roof materials and the progressive degradation of material due to weather conditions have to be taken into account. Depending on the cardinal direction, the roof materials might have natural differences in reflectance. Thus, the evaluation based on metrics that measure the width of the reflectance distribution can contain natural effects that lead to skewed results. However, the rBRDF and the relative biconical reflectance display an improvement compared to the manufacturer's reflectance values for most metrics. Therefore, the newly calculated reflectance displays an improvement under the assumption that the degradation of material is constant throughout each roof, and thus reflectance should be the same for the

individual roofs. Furthermore, in the case of Figure 3, we can see that there are outliers, but the general reflectance is still improved compared to the initial reflectance values (Table 3 and Figure 5). In summary, the reflectance calculation would benefit from further research under controlled conditions, which is beyond the scope of this contribution. However, we could demonstrate that the application of waveform-based angle of incidence estimation can enhance reflectance values for the rBRDF without the need for neighborhood information and that the utilization of the relative biconical reflectance shows a general improvement in reflectance values, compared to the initial reflectance values.

5.2 Waveform decomposition

From the comparison with the TLS dataset, two varying results of the new reflectance measures can be observed. There, for the roofs, the results align well with the general evaluation, while for the street data the results differ strongly. This is likely related to the differences in waveform, as seen in Figure 7 and further outlined in Rhomberg-Kauert et al. (2026). To overcome this future work should move from Gaussian decomposition towards a system response fitting (Schwarz and Pfennigbauer, 2018), where the emitted laser pulse including the ringing effect can be used, or explore other decompositions, e.g. heavy-tailed curves (Rhomberg-Kauert et al., 2026).

5.3 Angle of incidence methods

For reflectance measures that take into account the angle of incidence, the best angle of incidence estimation depends on the system type. For Sensor I, the waveform-based estimation outperforms the neighborhood-based angles, while for Sensor II it is vice versa. The difference between both results is probably related to the recorded waveforms, as for Sensor I the sampling rate is half the sampling rate of Sensor II. While Sensor I has a sampling interval of approximately 0.5 ns, compared to 1 ns for Sensor II, due to differences in components used by the manufacturer (RIEGL, Laser Measurement Systems GmbH, 2022b,a). Thus, extraction of the FWHM during the Gaussian fitting for Sensor II entirely fails for some points. The reduction of available points in combination with a lower waveform resolution limits the quality of waveform-based angle estimation for Sensor II. This can potentially be improved by fine-tuning the parameters of the method or achieving higher point density on the roofs, as, for example, in the TLS data.

Although the TLS data provides a secondary dataset with reflectance values based on significantly smaller footprint, due to the lower scan position of the TLS compared to the roofs there are still angle of incidence effects to be considered. Especially since the correction of TLS reflectance values is a re-occurring topic throughout the literature (Laasch et al., 2025). Thus, the usage of TLS data as reference remains limited and serves more as an outline to the improvement. Therefore, both the rBRDF and the biconical reflectance would benefit from a comparison to ground truth from radiometer measurements with wavelengths the same as the airborne laser scanners.

6. Conclusion

The study demonstrates that reflectance values acquired by airborne LiDAR, which utilize short laser pulses and a medium-sized footprint, can be improved based on the acquired data. The calculations based on the laser-radar equation and different

methods of incidence angle estimation reduced the MAD and IQR of the reflectance distributions for roofs in the study area. The angle-independent relative biconical reflectance presents the overall best results across both LiDAR systems that were investigated. However, the different scan patterns and challenges during the Gaussian fitting for Sensor II are also represented in the results. Overall, this leads to a smaller improvement for Sensor II. Future work should examine the reflectance measures presented in our study in a controlled environment and furthermore explore the impact of scan patterns on the differences in reflectance distributions. In conclusion, this study improved the calculation of reflectance values compared to standard processing reflectance values, where the results of the newly introduced reflectance calculation displayed lower MAD and IQR values for the same roof under varying angles of incidence.

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