Hierarchical Polygon-to-Point Collapsing for Multi-Scale Representation Based on the Straight Skeleton

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Abstract

This paper presents a LoD transition space for the dimensional collapse of a polygon into point(s) within a structured multi-scale framework. Unlike traditional cartographic generalisation, where topological relationships may be modified to improve map readability, the proposed method follows a model-based representation in which transitions are derived explicitly from straight skeleton events. The methodology uses the straight skeleton to generate a sequence of shrinking stages governed by edge and split events, each of which defines both a topological transformation and its corresponding geometric change. Based on these event-driven transitions, intermediate Levels of Detail (LoDs) are constructed and organized hierarchically. The resulting hierarchy is then mapped to the Dual Half-Edge (DHE) structure, where the primal space represents successive geometric states and the dual space represents their hierarchical relations along the scale dimension. This integration produces a unified 2D+1D representation that supports a continuous transition from polygon to point. In addition to its relevance for vario-scale cartography, the proposed framework has potential applicability in domains requiring structured shape transformation, such as animation and procedural modelling.

1. Introduction

Representing spatial features consistently across multiple scales has long been identified as a key challenge in Geographic Information Systems (GIS) and cartographic generalization, due to the need for both geometric and thematic adaptation when changing scale (Weibel and Dutton, 1999; Yuan, 2024). Traditional generalization approaches are often limited to discrete levels of detail (LoDs), resulting in abrupt transitions between map scales and potential topological inconsistencies. To overcome these limitations, *vario-scale representation* has emerged as a concept for continuous and structured transformations between LoDs (Meijers and van Oosterom, 2011; Van Oosterom, 2005; van Oosterom and Meijers, 2014). In this structure, 2D area objects are represented as 3D volumes (prisms), 1D line objects as 2D vertical faces and 0D point objects as 1D vertical lines (Van Oosterom et al., 2014). This paradigm requires models that preserve both geometric coherence and topological consistency during transformations from complex 2D geometries to simpler 1D or 0D forms in a 2D or 3D space. In traditional GIS, we use discrete Levels of Detail (e.g., LoD1, LoD2, LoD3). When you zoom in or out, the map "jumps" from one version of a polygon to another. Vario-scale representation (introduced primarily by van Oosterom) treats scale as a continuous dimension. Instead of separate files for each scale, all scales are stored in a single data structure where the geometry changes smoothly as the scale value changes.

Multi-scale representation provides the conceptual basis for vario-scale representation by describing how geographic data is systematically generalized across different scales. This process involves operations such as simplification, aggregation, merging, and typification, which reduce the geometric complexity of features while maintaining their essential spatial patterns (Li, 2006; Sester, 2005). For example, winding rivers may be simplified, building clusters typified, or adjacent land parcels merged into larger polygons to enhance map clarity. In traditional cartographic generalization, topological relationships are often altered or sacrificed through operations such as displacement, merging, or elimination to ensure visual legibility. In contrast, the proposed method aligns with model-based generalization principles. By integrating our straight-skeleton approach with the tGAP (topological Generalized Area Partition) structure, topological consistency is maintained automatically. The 2D+1D representation ensures that as a polygon collapses to a point, its spatial relationships with surrounding features are preserved within the volumetric partition, preventing gaps or overlaps that typically occur in purely cartographic workflows.

The straight skeleton (Aichholzer et al., 1996) provides an elegant geometric framework for hierarchical polygon decomposition. By simulating an inward wavefront propagation process, the straight skeleton generates a tree-like structure of monotone sub-polygons, each corresponding to intermediate stages of polygon contraction. The resulting polygon hierarchy captures the geometric relationships between the original boundary and its successive collapses, making it well-suited for map generalization, area reduction, and geometric simplification (Haunert and Sester, 2004; Meijers, 2016). Unlike the medial axis, which produces curved segments, the straight skeleton is composed entirely of straight-line edges, facilitating precise and computationally efficient implementations in GIS applications.

While the straight skeleton effectively defines the geometric hierarchy of shrinking polygons, managing the corresponding topological hierarchy across multiple LoDs requires a robust data structure. The Dual Half-Edge (DHE) structure (Boguslawski, 2011; Boguslawski and Gold, 2016) offers a suitable framework by explicitly linking geometric entities in the *primal space* with their topological duals in the *dual space*. Each cell, face, edge, and vertex in the primal representation has a dual counterpart, ensuring consistent and bidirectional navigation through all hierarchical levels. The DHE model’s pointer representation and navigation operators, e.g., *NextF* (next half-edge around a face), *PrevF* (previous half-edge), and *NextV* (around a shared vertex), provide efficient traversal and update mechanisms, enabling dynamic management of multi-scale structures. In the DHE structure, geometric and topological entities are represented through interconnected half-edges in both the primal and dual spaces. A *vertex* represents a 0D entity shared by adjacent half-

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edges, an *edge* consists of a symmetric pair of half-edges connecting two vertices (each belonging to a single face), and a *face* is a 2D surface bounded by a cyclic sequence of half-edges forming a closed loop. Once an edge is created, four corresponding edges are implicitly defined—two in the primal space (internal and external) and two in the dual space—providing consistent linkage between geometry and topology. The DHE structure thus maintains explicit connectivity between all geometric elements (faces, edges, and vertices) and provides the pointer representation between geometry (primal space) and topology (dual space)(Boguslawski, 2011). The DHE structure inherently represents hierarchical relationships through its primal–dual connectivity, making it well suited for constructing tree-like configurations that describe geometric transformations across multiple LoDs (Gholami et al., 2024).

Haunert and Sester. (2008) examined the use of the straight skeleton for collapsing area feature to centreline while considering neighbouring relationships; however, their work did not analyse how individual skeleton events affect neighbouring features throughout the generalization process. In contrast, the planned research will focus on capturing and analysing these event-based influences when collapsing area features to points within a hierarchical framework.

It is important to contextualize the proposed straight skeleton method within the broader field of dimensional collapse. Standard techniques often include centroid-based collapse, which is computationally efficient but primarily focuses on numerical averages of vertices , and ontology-driven dimensional collapse (Wolf, 2009), which relies heavily on semantic context and scale sensitivity. In contrast to these methods or common toolsets like FME’s CenterPointReplacer—which utilizes the center of a bounding box, the center of gravity while for a polygon with a hole the point representation can be outside of polygon, or an arbitrary internal point can be choose in FME, but the proposed approach offers a more "natural" simplification and define intermediate LoDs which is useful for a smooth transition. While traditional boundary simplification is highly dependent on manually assigned tolerance levels , the straight skeleton method ensures a smooth, continuous shrinking process that culminates in a point chosen by the final geometric event of the skeleton. This provides a superior foundation for vario-scale representations and smooth transitions in applications like 2D animation and progressive data transfer.

Gholami et al. (2025) integrate the straight skeleton, DHE structure, and Space-Scale Cube (SSC) into a conceptual framework for *2D-to-1D and 0D generalization*. The current paper provides the first rigorous mathematical and algorithmic implementation of this theory. Specifically, this work introduces a novel method for calculating the offset distance (r) required to generate intermediate LoDs. Furthermore, this paper details the practical mapping of these geometric events into the DHE structure, providing a functional data model that was absent in the previous conceptual introduction. Thus, this paper moves beyond the 'why' of the method to provide the 'how' through concrete implementation and result validation.

The present study addresses this gap by proposing a hierarchical collapsing model that transforms a 2D polygon into a single 0D feature through a sequence of controlled shrinking operations derived from the straight skeleton. Each shrinking stage is treated as a distinct multi-scale state, while the parent–child relations between successive states are organized as a hierarchical structure. This hierarchy is subsequently embedded into the DHE data structure, where the primal space represents the geometric states and the dual space expresses their hierarchical connectivity along the scale dimension.

The main contributions of this paper are threefold:

1. Development of a hierarchical polygon-to-point collapsing algorithm from a generalisation perspective, based on straight-skeleton geometry and event-driven shrinking.
2. Construction of intermediate multi-scale states derived directly from straight skeleton events, allowing the full polygon-to-point transition to be represented explicitly.
3. Mapping of the resulting hierarchy to the DHE structure, establishing a direct correspondence between geometric states in the primal space and hierarchical relations in the dual space.

One practical example of this process is the gradual collapsing of island polygons during map generalisation. As scale decreases, an island may progressively transform into simplified intermediate geometries before ultimately being represented as a single point. Depending on the polygon shape, intermediate stages may involve both splitting and collapsing operations.

In addition to the hierarchical collapsing process, this paper presents a preliminary vario-scale visualization of the resulting 2D+1D representation. In this interpretation, the scale dimension is treated as the third axis, and the full transition is represented as a **LoD Transition Space (LTS) of Polygon-to-point collapsing**. Within this space, topological changes occur through straight skeleton events, and the corresponding geometric transformations can be traced continuously from the original polygon to its final point representation.

The following sections describe the methodology for constructing the hierarchical polygon-to-point model (Section 2), demonstrate its implementation and results (Section 3), and discuss its implications for continuous-scale spatial representations (Section 4).

2. Methodology and Implementation

The proposed methodology constructs a hierarchical polygon-to-point collapsing model by simulating the inward contraction of a polygon through the straight skeleton process. Each shrinking step corresponds to a new state in the multi-scale hierarchy, and the transition between successive states is governed by straight skeleton events. In this framework, topology changes first through event occurrence, and the resulting geometry is then updated accordingly. The hierarchical relations between these states are represented using the Dual Half-Edge (DHE) structure, which provides explicit linkage between geometric representation and hierarchical organization. The overall workflow is shown in Figure 1 and consists of three main stages:

1. polygon initialization in the DHE structure,
2. detection of straight skeleton events, and
3. hierarchical structure generation and scale assignment.

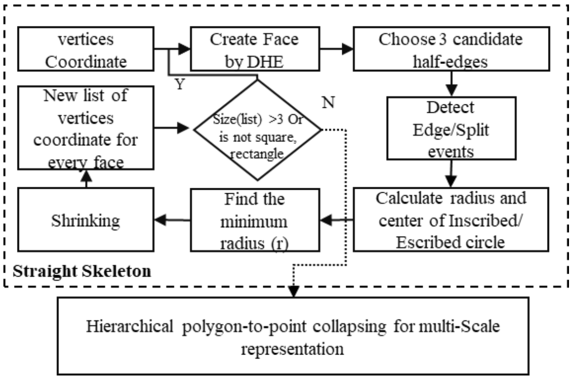


Figure 1. Research methodology

2.1 Polygon Initialization in the DHE Structure

The process begins with a polygon defined by an ordered list of vertex coordinates $P = \{v_0, v_1, \dots, v_n\}$, where vertices are sorted in counterclockwise order. Each polygon is represented as a face constructed using the DHE structure and *MakeFace* operator (Boguslawski, 2011; Gholami et al., 2024). This structure serves as the computational foundation for detecting skeleton events and managing the geometric contraction process. Each edge of the polygon is treated as a half-edge e_i, and its symmetric counterpart e_i^S defines adjacent relationships between neighboring faces in one feature.

2.2 Detection of Skeleton Events

The hierarchical collapsing process is governed by two key event types derived from straight-skeleton theory (Aichholzer et al., 1996; Huber and Held, 2012):

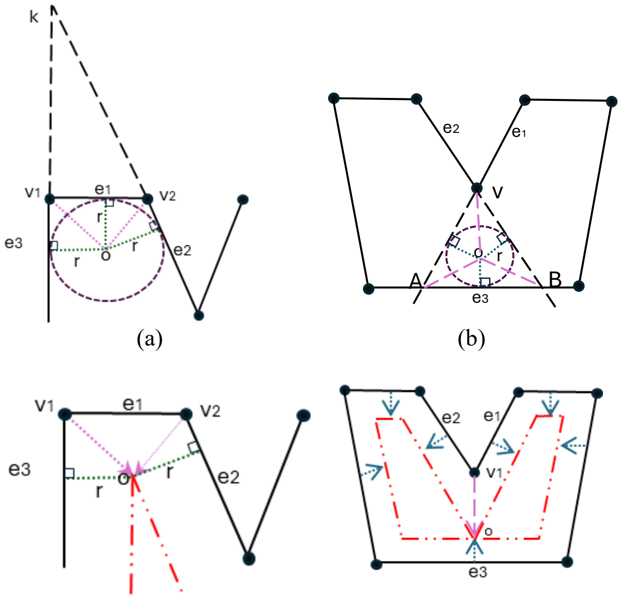
1. Edge Event – occurs when an edge of the polygon contracts to zero length, causing two adjacent edges to meet at a single point.
2. Split Event – occurs when a reflex vertex (with an interior angle greater than 180°) intersects an opposite edge, dividing the polygon into two sub-polygons.

In each iteration, candidate half-edges are selected for event detection by traversing the polygon using DHE navigation. For an edge event, three consecutive half-edges (e_1, e_2, e_3) are evaluated. The intersection of the angular bisectors of e_1 and e_2^S is computed, and if the intersection point lies inside the polygon, an edge event is recorded.

For a split event, a reflex vertex is first identified by measuring the internal angle between consecutive edges. If the angle exceeds 180°, the vertex is treated as a split candidate. Using DHE traversal around the face, potential intersections between the reflex vertex bisector and opposite edges are tested. If an intersection exists, a split event is registered. Each detected event is associated with a corresponding offset distance r, which is defined as the radius of the inscribed or escribed circle of the triangle formed by the intersecting edges. The event with the minimum r determines the next collapsing step.

Figure 2 illustrates the geometric configuration of the edge and split events and their corresponding offset distances. After computing the minimal r, all polygon edges are offset inward by distance r. The intersection of the offset lines defines the new vertices of the shrunken polygon. If the new polygon degenerates into a line or a point, it marks the end of the current hierarchical branch. Otherwise, the resulting polygon is stored as a child node of the previous polygon in the hierarchy.

The split and edge events before and after the shrinking process are shown in Figure 2. The edge event extended e_2 and e_3 lines with e_1 line have formed a V_1KV_2 triangle, and the center (o) of the escribed circle to the e_1 side of the triangle is that intersection of v_1 and v_2 angular bisectors, this o is that edge event point, therefore we can calculate the r, and Figure 2b the split event the extended e_1 and e_2 lines with e_3 line segment formed an AVB triangle and the center (o) of the inscribed circle is that intersection of A, V, and B angular bisectors, this o is that split event point, therefore we can calculate the r.



(a) (b) (c) (d)

Figure 2. (a) edge event; (b) split event; (c) shrinking by edge event; (d) shrinking by split event.

As shown in Figure 2c the e_1 edge has vanished in o after the shrinking and in Figure 2d after shrinking by split event the face is divided into two small faces.

If all resulting child faces at the current stage have collapsed to vertices, the process terminates. The final output is therefore not only a point representation, but also the complete polygon hierarchy derived from the straight skeleton. Since each shrinking step is associated with a specific event distance, the sequence of hierarchy generation can also be interpreted along the scale dimension.

2.3 Hierarchical Structure Generation and Scale Assignment

The hierarchical collapsing process produces a polygon hierarchy, where each LoD corresponds to a smaller, shrunken polygon generated from its parent. The initial polygon represents the root node (P_0), and each shrinking iteration adds a new LoD ($P_1, P_2, \dots, P_n$), until the final polygon collapses into a 0D point. Each node in this hierarchy stores geometric information (vertex coordinates, edges, and faces) as well as topological connections to its parent and child nodes.

The hierarchical tree can be defined as:

$$H = \{P_i \mid P_{i+1} = f(P_i, r_i)\}, \qquad i = 0, 1, \dots, n-1$$

where f (P_i, r_i) represents the shrinking operation based on the minimal offset distance r_i. The process terminates when P_n becomes a vertex or when no further events can be detected.

To establish a continuous transition across levels, each hierarchy level is associated with a LoD value (LoD_l). Two strategies are used for assigning scales:

1. Uniform:

$$LoD_l = LoD0 + \frac{(LoD_n - LoD_0)}{n}, l = \{1,2,..,n-1\} \qquad (1)$$

2. Offset-based:

$$LoD_l = LoD_0 + \frac{\sum_{i=0}^{l} r_i}{\sum_{i=0}^{n} r_i} \times (LoD_n - LoD_0), l = \{1,2,..,n-1\} \qquad (2)$$

where LoD_l = LoD of level l
r_i= minimum r of level i
LoD_n= Lodi of level n
LoD_0= LoD of level 0

Within the context of polygon-to-point collapsing, LoD_0 and LoD_n represent the initial polygon and the final point geometry, respectively. Each level l between these two states signifies an intermediate LoD in the transformation.

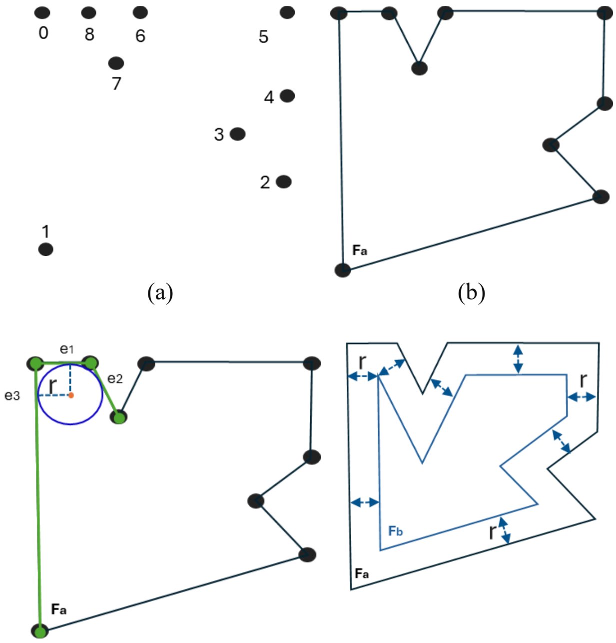
We clarify that in a vario-scale environment, the absolute number of levels for an individual polygon is secondary to its representation in the continuous scale dimension. While different polygons possess varying geometric complexities (resulting in a different number of straight skeleton events or 'levels'), they are all mapped to a universal scale range [S_{min}, S_{max}]. By using the formalizations in Equations 1 and 2, the 'birth' and 'death' of each feature are synchronized along the Z-axis (the scale axis). Therefore, navigating between scales is not a matter of jumping between discrete levels, but rather of slicing the 2D+1D space-scale partition at a specific Z value. This ensures that all polygons in a scene—regardless of their internal complexity—are rendered at the appropriate state for that specific global scale.

The integration of these hierarchical and topological relationships is visualized in Section 3.2, where the resulting DHE-based vario-scale representation illustrates the correspondence between the geometric and hierarchical layers.

From a process perspective, the proposed framework represents a sequence of event-driven transitions in which topology and geometry evolve together. Straight skeleton events, particularly edge and split events, define the moments at which the polygon structure changes, while the corresponding shrinking operation updates the geometry of the affected feature. This repeated interaction between topological transformation and geometric contraction continues until the polygon is fully reduced to a point. In the context of this study, the resulting integrated 2D+1D representation is interpreted as a LoD Transition Space (LTS), where the complete dimensional collapse of a feature can be traced across scale.

3. Implementation and Results

The proposed method was evaluated using two distinct test cases. The first polygon is based on coordinates from Aichholzer et al. (1996), providing a benchmark consistent with established straight skeleton literature. The second is an arbitrary mock polygon specifically designed to demonstrate the mechanics of dimensional collapse and the resulting intermediate levels of detail (LoDs). While these cases serve to illustrate the core principles of the 2D+1D representation, the algorithm is robust and capable of processing significantly more complex polygonal geometries found in real-world spatial data.
The input vertices are shown in Figure 3a. Then the first face F_a is created and selected as the root of a tree structure (Figure 3b). Navigating around the face, edge and split events are identified by *Edgeevent* and *Splitevent* operators. There are 5 edge events and 2 split events detected in this face with the minimum r at the level 0 shown in Figure 3c. Based on r the F_a is shrunk (Figure 3d). The output feature F_b is a new face and it is stored as child node of F_a in a polygon hierarchy tree structure (Figure 6b). Thus, the second level of the polygon hierarchy tree structure is formed.



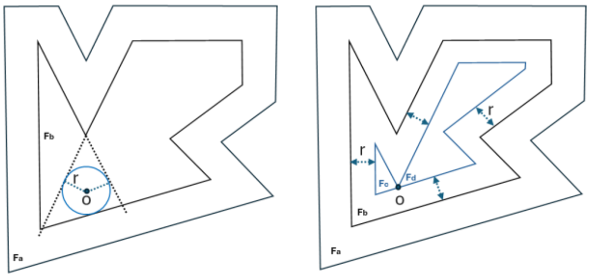
(a) (b)

(c) (d)

Figure 3. Methodology process for level 0: (a) input vertices; (b) a face F_a (c); finding the minimum r; (d) shrinking of F_a by r.

A sub-tree is created for this child at this level and the process will be repeated for this sub-tree. Since the last event was an edge event, this level contains only one child, resulting in the creation of a single sub-tree.

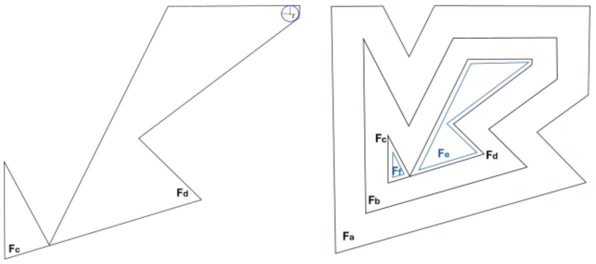
There is only a single face at this level. Four edge events and two split events are detected in the face, with the minimum r from a split event, as shown in Figure 4a. Based on this r, F_b undergoes shrinking (Figure 4b). Due to the split event, two new faces, F_c and F_d, are created and stored as children of F_b in the polygon hierarchy tree structure.



(a) (b)

Figure 4. Methodology process for level 1: (a) finding the minimum r; (b) the shrinking of F_b by r. All arrows denote the propagation distance r, which is maintained at a constant length to ensure a uniform offset.

There are two faces at level 2, F_c and F_d. All split and edge events are detected, with the minimum r at this level is an edge event (Figure 5a). Based on this r, the shrinking step is applied to all faces at this level, as shown in Figure 5b.



(a) (b)

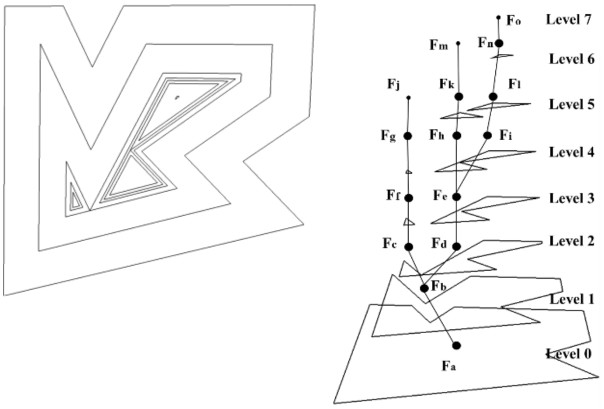
Figure 5. Methodology process for level 2: (a) finding the minimum *r*; (b) The shrinking of *Fc* and *F$_d$* by *r*.

When a feature node corresponds to a regular polygon such as a triangle, square, or rectangle, no additional split or edge events need to be detected. In these cases, the shrinking process terminates naturally, and the resulting child node is represented by a single point whose coordinates correspond to the centroid of the polygon. The value of r in this case is defined as the distance between the centroid and any vertex of the polygon. When the feature node is a parallelogram, the next shrinking step produces a line feature. If the ultimate goal is a complete collapse to a point, this line is subsequently reduced to a single vertex representing the final stage of the hierarchical polygon-to-point collapsing process. In this case, r is equal to half the length of the resulting line segment. However, if the minimum r is detected from another face, the shrinking of the given feature node may not immediately result in a 0D feature at the next level. For example, in Figure 5b at level 2, the child of F_c is a shrunk triangle (F_f), whereas in Figure 6b at level 4, the child of F_g is a 0D feature. This process continues at each level until the final child (or children) consists solely of 0D features.

3.1 Multi-scale representation and hierarchical tree

Figure 6 illustrates the final polygon hierarchy, with the corresponding hierarchical tree structure attached in Figure 6b. Each level of this tree structure represents a scaled map level in the multi-scale representation.

The 2D polygon hierarchy in this paper differs from that of Aichholzer et al. (1996), as their approach focuses only on the transition from 2D to 1D. In contrast, this study extends the process further, shrinking all children at each level until the final child becomes a single vertex (2D to 0D transition).



(a) (b)

Figure 6. Polygon Hierarchy: (a) 2D visualization, (b) multi-scale representation and its corresponding hierarchy structure. The F_x labels are positioned at the mid-points between successive levels to denote the transition phases.

3.2 Vario-scale representation

The vario-scale representation, together with its corresponding hierarchical tree structure, can be constructed using the DHE construction operators—such as *MakeEdge*, *MakeFace*, and *JoinByFace*—as introduced by Boguslawski (2011). These operators establish the geometric and topological relationships required to organize successive polygon-to-point collapsing stages within a unified structure.

Figure 7 illustrates how the hierarchical tree derived from the straight skeleton is mapped into the DHE framework through dual connections and dual vertices. This mapping produces a topologically connected representation in which:

- The primal space represents the vario-scale layers obtained from successive shrinking steps, and
- The dual space captures the hierarchical relationships linking these layers across the scale dimension.

Through this integration, the DHE model provides a consistent framework for visualizing and analyzing the hierarchical polygon-to-point collapsing process as a continuous volumetric vario-scale structure.

Unlike the classical “roof model” interpretation of the straight skeleton proposed by Aichholzer et al. (1996) and developed by (Held and Palfrader, 2017; Sugihara, 2019; Sugihara and Hayashi, 2008), where the third dimension represents the physical height of a building roof, the 3D visualization in this study employs the third dimension as a *scale dimension*. Consequently, the resulting 3D structure represents not the elevation of objects but their progressive generalization across multiple levels of detail. The output of this process forms a SSC, referred to as a *vario-scale representation*, where each layer corresponds to a specific stage in the polygon-to-point collapsing process. In future research, the proposed framework will be extended to include neighbouring features to examine how their spatial relationships evolve during generalization within the vario-scale model. Moreover, DHE construction operations will be formally defined to build the full vario-scale representation of this hierarchical collapsing process.

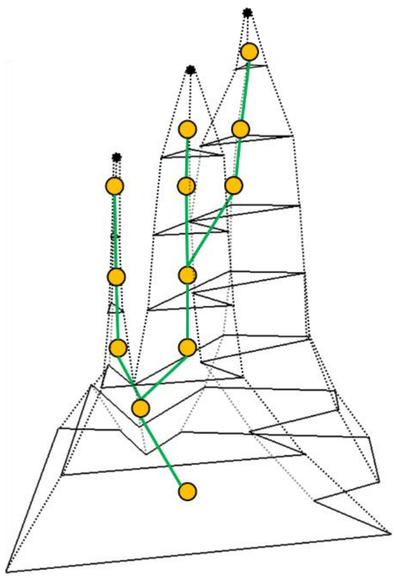


Figure 7. Vario-scale representation and its hierarchical tree structure constructed using the DHE structure. In the primal space, solid black lines represent the spatial geometry (2D and

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0D features), while dotted lines indicate scale transitions (1D connections). In the dual space, orange circles denote dual vertices, and solid green lines illustrate dual connections between corresponding dual vertices.

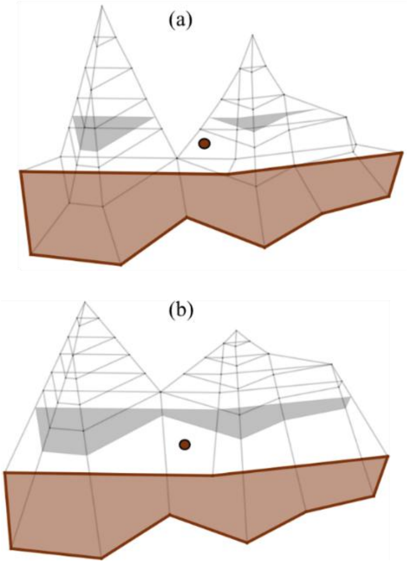

Figure 8. Vario-scale representation of another arbitrary polygon under different scale assignment strategies: (a) uniform distribution and (b) offset-based distribution. The original polygon is depicted in brown, while the gray polygons represent the intermediate level generated by slicing the SSC at the same horizontal xy-plane.

In Figure 7, each dual vertex represents a 3D cell. These cells can be merged along their shared horizontal faces to form a unified 2D+1D representation, where the 2D domain defines the spatial extent and the 1D dimension encodes topological changes across the scale axis. This integrated space constitutes a vario-scale representation. A specific LoD can be extracted as a snapshot by slicing this 2D+1D structure with a horizontal *xy*-plane at a given height (e.g., $Z = LoD_l$). While non-horizontal slicing surfaces can produce mixed-scale maps (Van Oosterom et al., 2014), a horizontal slice in this polygon-to-point collapsing process may occasionally yield multiple geometric primitives.

However, the fundamental property of this method is that it maintains a single, continuous 3D feature; the appearance of multiple features is merely an artifact of the slicing process. To ensure semantic and topological consistency when multiple polygons arise, three solutions are proposed:

1. **Attribute Propagation:** Copy the attributes of the original polygon to all fragmented polygons created during the slice.
2. **Structural Connectivity:** Connect individual polygons using 'bridge edges' derived from the straight skeleton.
3. **Geometric Adjustment:** Normalize the skeleton nodes to the same height to eliminate 'valleys' between peaks. This ensures the polygon collapses into a single multiline structure before reaching its final destination as the highest peak (point).

Figure 8 illustrates the proposed smooth transition for an arbitrary polygon using two distinct strategies. In the uniform distribution (Figure 8a), the slopes of the inclined transition faces vary. In contrast, the offset-based distribution (Figure 8b) maintains a constant slope across all transition faces, consistent with the standard 'roof model' gradient (slope=1). When both structures are sliced by the same horizontal plane at LoD_l, they yield different geometric outputs, as demonstrated in Figure 8. Notably, the topological evolution in offset-based distribution provides a more natural simplification. While our 2D+1D representation shares geometric similarities with the straight skeleton 'roof model,' it differs fundamentally in its interpretation of the third dimension. In this vario-scale framework, the Z-axis represents the scale dimension, and the structure contains pre-defined intermediate LoDs corresponding to specific straight skeleton events.

Beyond its primary utility in vario-scale cartography and continuous generalization, the output of this model is highly adaptable to other domains such as computer animation, procedural gaming, and medical imaging, where smooth geometric interpolation is essential. Accordingly, the resulting integrated 2D+1D representation is interpreted in this study as a LoD Transition Space (LTS), in which the polygon-to-point transformation can be traced through successive topological and geometric states.

4. Discussion and Conclusion

This study presented a hierarchical approach for collapsing 2D polygons into single points within a structured multi-scale framework and demonstrated how the resulting hierarchy can be represented through the DHE model. The proposed method combines the geometric behaviour of the straight skeleton with the topological organization provided by the DHE structure. By integrating these two components, the framework captures both the shrinking sequence of polygon geometry and the hierarchical relations between successive scale states.

A key characteristic of the proposed approach is that the collapsing process is driven by straight skeleton events. Edge and split events define the moments at which topological changes occur, and these changes are subsequently reflected in the geometry through inward shrinking. As a result, the transformation from polygon to point is not treated as a single discrete operation, but as a traceable sequence of intermediate states. This makes the approach particularly suitable for structured multi-scale representation, where transitions between levels should remain interpretable and spatially coherent.

The study also showed how the resulting polygon hierarchy can be visualized in a 2D+1D framework, where the third dimension represents scale rather than physical height. In this interpretation, the resulting representation can be understood as a LoD Transition Space (LTS), in which both topological and geometric changes are expressed within a unified structure. This offers a foundation for continuous polygon-to-point transition rather than abrupt switching between discrete representations.

Beyond its relevance to vario-scale cartography and model-based generalisation, the proposed framework may also be applicable in domains where gradual shape transformation is required, such as animation, procedural modeling, and related geometric applications. Future work will focus on extending the approach to neighboring features, polygons with holes, and more complex real-world datasets, as well as investigating how such transitions can be integrated into broader vario-scale representations.

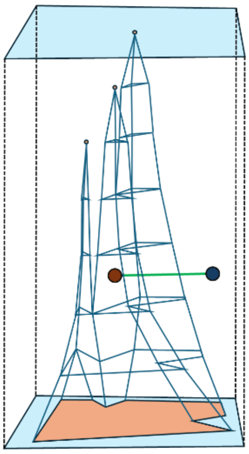

Figure 9. Vario-scale representation of an island and sea features during polygon-to-point collapsing. In the dual space, brown and solid blue circles denote island and sea dual vertices, respectively, and solid green lines illustrate dual connections between features.

A practical example of this process is the smooth collapsing of an island polygon during map generalization, where the island feature is gradually simplified and eventually represented as a single point at smaller scales. Figure 9 illustrates a practical example of the vario-scale representation of island and surrounding sea features during the polygon-to-point collapsing process.

The practical utility of the proposed straight skeleton-based collapsing method lies in its ability to provide a geometrically 'natural' transition, which is often lacking in standard GIS tools. While common operations like the FME offer computational efficiency, they result in a sudden, discrete jump from a 2D area to a 0D point. Our 2D+1D vario-scale framework bridges this gap, offering a foundation for continuous cartographic generalization. By treating the scale as a third dimension, we move beyond static map layers toward a dynamic environment where features evolve fluidly. This is particularly relevant for National Mapping Agencies (NMAs) and developers of web-based GIS, as it supports progressive data transfer—allowing the system to stream only the necessary geometric detail based on the user's zoom level, thereby optimizing bandwidth without sacrificing topological consistency.

In conclusion, this research demonstrates that the straight skeleton provides an effective mechanism for hierarchical polygon-to-point collapsing within a structured multi-scale framework. The resulting output of this study is formulated as a Smooth Transition Space (STS), representing a 3D polygon-to-point transition in which both topological and geometric changes can be traced across the scale dimension. Within this space, straight skeleton events define the moments of structural transformation, while the corresponding shrinking process progressively modifies the geometry until the polygon is fully reduced to a point. This approach follows the internal configuration of the polygon and produces transitions that are spatially coherent and visually smooth.

Beyond its relevance to vario-scale cartography, the proposed framework also has potential applicability in fields such as computer graphics, animation, and procedural modeling, where gradual geometric transformation is important. Future work will focus on extending this 2D+1D representation to more complex real-world datasets, including neighboring features, polygon-to-line, polygon-to-polygon and multiline-to-multiline transitions, as well as investigating how semantic constraints may support the selection of the final collapse point.

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References

Aichholzer, O., Aurenhammer, F., Alberts, D., Gärtner, B., 1996. A Novel Type of Skeleton for Polygons, in: Maurer, H., Calude, C., Salomaa, A. (Eds.), J.UCS The Journal of Universal Computer Science. Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 752–761. https://doi.org/10.1007/978-3-642-80350-5_65

Boguslawski, P., 2011. Modelling and analysing 3D building interiors with the dual half-edge data structure.

Boguslawski, P., Gold, C., 2016. The dual half-edge—A topological primal/dual data structure and construction operators for modelling and manipulating cell complexes. ISPRS Int. J. Geo-Inf. 5, 19.

Gholami, A., Boguslawski, P., Meijers, M., van Oosterom, P., 2025. 2D to 1D and 2D to 0D Geometry Generalization Based on the Straight Skeleton by the Dual-Half Edge Structure. Int. Arch. Photogramm. Remote Sens. Spat. Inf. Sci. 48, 513–519.

Gholami, A., Boguslawski, P., Meijers, M., van Oosterom, P., 2024. Evaluation of the dual half-edge data structure for implementation of a vario-scale model. Int. Arch. Photogramm. Remote Sens. Spat. Inf. Sci.-ISPRS Arch. 48, 25–32.

Haunert, J.-H., Sester, M., 2008. Area Collapse and Road Centerlines based on Straight Skeletons. GeoInformatica 12, 169–191. https://doi.org/10.1007/s10707-007-0028-x

Haunert, J.-H., Sester, M., 2004. Using the straight skeleton for generalisation in a multiple representation environment, in: ICA Workshop on Generalisation and Multiple Representation, Leicester.

Held, M., Palfrader, P., 2017. Straight skeletons with additive and multiplicative weights and their application to the algorithmic generation of roofs and terrains. Comput.-Aided Des. 92, 33–41.

Huber, S., Held, M., 2012. A FAST STRAIGHT-SKELETON ALGORITHM BASED ON GENERALIZED MOTORCYCLE GRAPHS. Int. J. Comput. Geom. Appl. 22, 471–498. https://doi.org/10.1142/S0218195912500124

Li, Z., 2006. Algorithmic foundation of multi-scale spatial representation. CRC press.

Meijers, B.M., van Oosterom, P.J.M., 2011. THE SPACE-SCALE CUBE: AN INTEGRATED MODEL FOR 2D POLYGONAL AREAS AND SCALE. Int. Arch. Photogramm. Remote Sens. Spat. Inf. Sci. XXXVIII-4-C21, 95–102. https://doi.org/10.5194/isprsarchives-XXXVIII-4-C21-95-2011

Meijers, M., 2016. Building simplification using offset curves obtained from the straight skeleton, in: Proceedings of the 19th

ICA Workshop on Generalisation and Multiple Representation. Helsinki, Finland. pp. 1–18.

Sester, M., 2005. Optimization approaches for generalization and data abstraction. Int. J. Geogr. Inf. Sci. 19, 871–897. https://doi.org/10.1080/13658810500161179

Sugihara, K., 2019. Straight skeleton computation optimized for roof model generation, in: Proceedings of the WSCG.

Sugihara, K., Hayashi, Y., 2008. Automatic generation of 3D building models with multiple roofs. Tsinghua Sci. Technol. 13, 368–374.

Van Oosterom, P., 2005. Variable-scale Topological Data Structures Suitable for Progressive Data Transfer: The GAP-face Tree and GAP-edge Forest. Cartogr. Geogr. Inf. Sci. 32, 331–346. https://doi.org/10.1559/152304005775194782

Van Oosterom, P., Meijers, M., Stoter, J., Šuba, R., 2014. Data Structures for Continuous Generalisation: tGAP and SSC, in: Burghardt, D., Duchêne, C., Mackaness, W. (Eds.), Abstracting Geographic Information in a Data Rich World, Lecture Notes in Geoinformation and Cartography. Springer International Publishing, Cham, pp. 83–117. https://doi.org/10.1007/978-3-319-00203-3_4

van Oosterom, P., Meijers, M., 2014. Vario-scale data structures supporting smooth zoom and progressive transfer of 2D and 3D data. Int. J. Geogr. Inf. Sci. 28, 455–478. https://doi.org/10.1080/13658816.2013.809724

Weibel, R., Dutton, G., 1999. Generalising spatial data and dealing with multiple representations. Geogr. Inf. Syst. 1, 125–155.

Wolf, E.B., 2009. Ontology-Driven Generalization of Cartographic Representations by Aggregation and Dimensional Collapse, in: Bernstein, A., Karger, D.R., Heath, T., Feigenbaum, L., Maynard, D., Motta, E., Thirunarayan, K. (Eds.), The Semantic Web - ISWC 2009, Lecture Notes in Computer Science. Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 990–997. https://doi.org/10.1007/978-3-642-04930-9_65

Yuan, M., 2024. Multiple representations in geospatial databases, the brain’s spatial cells, and deep learning algorithms. Cartogr. Geogr. Inf. Sci. 51, 687–701. https://doi.org/10.1080/15230406.2023.2264758